Skolverket

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MAT

Mathematics

Mathematics has a history stretching back many thousands of years with contributions from many cultures. It has developed not only out of practical necessity, but also as a result of people's curiosity and desire to explore mathematics as an end in itself. Communication using the language of mathematics is similar all over the world. As information technology develops mathematics is being used in increasingly complex situations. Mathematics is also a tool in science and different professions. Ultimately mathematics is about discovering patterns and formulating general relationships.

Aim of the subject

Teaching in mathematics should aim at students developing their ability to work mathematically. This involves developing an understanding of mathematical concepts and methods, as well as different strategies for solving mathematical problems and using mathematics in social and professional situations. Teaching should give students the opportunity to challenge, deepen and broaden their creativity and mathematical skills. In addition, it should contribute to students developing the ability to apply mathematics in different contexts, and understand its importance for the individual and society.
Teaching should cover a variety of working forms and methods of working, where investigative activities form a part. Where appropriate, teaching should take place in environments that are relevant and closely related to praxis. Teaching should give students the opportunity to communicate using different forms of expression. In addition, it should provide students with challenges, as well as experience in the logic, generalisability, creative qualities and multifaceted nature of mathematics. Teaching should strengthen students' confidence in their ability to use mathematics in different contexts, and provide scope for problem solving both as a goal and an instrument. Teaching should also give students the opportunity to develop their ability to use digital technology, digital media, and other tools which can occur in subjects typical of programmes.

Teaching in mathematics should give students the opportunity to develop their ability to:

1) use and describe the meaning of mathematical concepts and their interrelationships.
2) manage procedures and solve tasks of a standard nature with and without tools.
3) formulate, analyse and solve mathematical problems, and assess selected strategies, methods and results.

4) interpret a realistic situation and design a mathematical model, as well as use and assess a model's properties and limitations.
5) follow, apply and assess mathematical reasoning.
6) communicate mathematical thinking orally, in writing, and in action.
7) relate mathematics to its importance and use in other subjects, in a professional, social and historical context.

Courses in the subject

- Mathematics 1a, 100 credits, which builds on knowledge from the compulsory school or equivalent. Grades in the course cannot be included in the student's diploma together with grades in the courses, mathematics 1b or 1c. The course is included in all vocational programmes.
- Mathematics 1b, 100 credits, which builds on knowledge from the compulsory school or equivalent. Grades in the course cannot be included in the student's diploma together with the grades in the courses, mathematics 1a or 1c. The course is included in the Business Management and Economics Programme, the Arts Programme, the Humanities Programme, and the Social Science Programme.
- Mathematics 1c, 100 credits, which builds on knowledge from the compulsory school or equivalent. Grades in the course cannot be included in the student's diploma together with grades in the courses, mathematics 1a or 1b. The course is included in the Natural Science Programme and the Technology Programme.
- Mathematics 2a, 100 credits, which builds on the course mathematics 1a. Grades in the course cannot be included the student's diploma together with grades in the courses, mathematics 2b or 2c.
- Mathematics 2a, 100 credits, which builds on the course mathematics 1b. Grades in the course cannot be included in the student's diploma together with grades in the courses, mathematics 2a or 2c.
- Mathematics 2c, 100 credits, which builds on the course mathematics 1c. Grades in the course cannot be included in student's diploma together with grades in the courses, mathematics 2a or 2b.
- Mathematics 3b, 100 credits, which builds on the courses, mathematics 2a or 2b. Grades in the course cannot be included in the student's diploma together with the grade in the course mathematics 3c.
- Mathematics 3c, 100 credits, which builds on the courses, mathematics 2a or 2c. Grades in the course cannot be included in the student's diploma together with the grade in the course mathematics 3b.

- Mathematics 4, 100 credits, which builds on the courses, mathematics 3b or 3c.
- Mathematics 5, 100 credits, which builds on the course mathematics 4.
- Mathematics – specialisation, 100 credits, which builds on the course mathematics 4, and can be studied a number of times with different contents.

MATMAT01a

Mathematics 1a

The course mathematics 1a covers points 1–7 under the heading Aim of the subject. The course covers basic knowledge in the subject.

Core content

Teaching in the course should cover the following core content:

Understanding of numbers, arithmetic and algebra

- Methods of calculating using real numbers in different forms in daily life and in subjects typical of a programme, including rough approximation, mental arithmetic and estimation, as well as strategies for using digital tools.
- Strategies for using tools from subjects typical of a programme, such as forms, templates, rules of thumb, regulations, manuals and handbooks.
- Handling algebraic expressions and formulae relevant in subjects typical of a programme, as well as methods for solving linear equations.

Geometry

- Properties and representations of geometric objects, such as drawings, practical designs, and coordinate systems.
- Geometric concepts chosen with regard to the needs of subjects typical of a programme, such as scale, vectors, uniformity, congruence, sine, cosine, tangent and symmetries.
- Methods of measuring and calculating quantities that are crucial in subjects typical of programmes.
- Units, unit conversions and processing of numerical values which are crucial in subjects typical of programmes, and rounding off methods relevant to subjects typical of programmes.

Relationships and change

- Advanced percentage concepts: per mille, ppm and percentage points.
- The concepts of rate of change and index, as well as methods for calculating interest and amortisation for different types of loans.
- The concepts of ratio and proportionality in reasoning, calculations, measurements and designs.
- Differences between linear and exponential processes.

Probability and statistics

- Descriptive statistics using spreadsheets, and examination of how statistical methods and results are used in society and professional life.

- The concepts of dependent and independent events, as well as methods for calculating probabilities in multi-stage random trials, using examples from games, and risk and safety assessments.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- How mathematics can be used as a tool in dealing with wide-ranging problem situations in subjects typical of a programme. The opportunities and limitations of mathematics in these situations.
- Mathematical problems relevant to personal finances, societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **with some certainty** show the meaning of key concepts in action, and **in basic terms** describe their meaning using **some** other representation. In addition, students switch **with some certainty** between these representations. Students can **with some certainty** use concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations.** In their work students handle **some simple** procedures, identify errors and solve standard tasks **with some certainty**, both with and without digital and other appropriate tools.

Students can formulate, analyse and solve practical mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform appropriate parts of problem situations in subjects typical of a programme into mathematical formulations by **informally** applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.

Students can apply **simple** mathematical reasoning and in a **simple** assessment evaluate their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, simple written form and action **with elements of** mathematical representations.

By giving examples students relate something in **the course content** to its importance in professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **with some certainty** show the meaning of key concepts in action, and **in detail** describe their meaning using **some** other representations. In addition, students switch **with some certainty** between these representations. Students can **with some certainty** use concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, identify **and correct** errors and solve standard tasks **with certainty**, both with and without digital and other appropriate tools.
Students can formulate, analyse and solve practical mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform appropriate parts of problem situations in subjects typical of a programme into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and in a **balanced** assessment evaluate their own reasoning and that of others, and differentiate between guesses and well founded statements. In addition, students can express themselves **with some certainty** in speech, simple written form and action **and use** mathematical **symbols and other** representations **with some adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **with certainty** show the meaning of key concepts in action and **in detail** describe their meaning using **several** other representations. In addition, students are able to switch **with certainty** between these different representations. Students can **with certainty** use concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, discover **and correct** errors, and solve standard tasks **with certainty and effectively,** both with and without digital and other appropriate tools.
Students can formulate, analyse and solve practical mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in rhetorical algebra**. In their work students re-express and transform appropriate parts of problem situations in subjects typical of a programme into mathematical formulations by **choosing**, applying **and adapting** mathematical models. Students can in a **balanced** assessment evaluate the plausi-

bility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning and in a **balanced** assessment evaluate their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with certainty** in speech, simple written forms and in action **and use** mathematical **symbols and other** representations **with good adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT01b

Mathematics 1b

The course mathematics 1b covers points 1–7 under the heading Aim of the subject. The course covers basic knowledge in the subject.

Core content

Teaching in the course should cover the following core content:

Understanding of numbers, arithmetic and algebra

- Properties of a range of whole numbers, different number bases, and the concepts of prime numbers and divisibility.
- Methods of calculating in everyday life and for subjects typical of programmes, real numbers written in different forms, including powers with integer exponents, and strategies for using digital tools.
- Processing of algebraic expressions and formulae relevant to subjects typical of programmes.
- The concept of linear inequality.
- Algebraic and graphical methods for solving linear equations and inequalities and exponential equations.

Geometry

- The concept of symmetry and different types of symmetry transformations of figures in a plane, and the occurrence of symmetry in nature and the arts from different cultures.
- Representations of geometric objects and symmetries through words, practical designs and aesthetic expressions.
- Mathematical reasoning using basic logic, including implication and equivalence, and comparisons with how to reason in everyday contexts and in different subject areas.
- Illustration of the concepts of definition, theorem and proof, such as the Pythagorean theorem and the sum of the angles of a triangle.

Relationships and change

- Advanced percentage concepts: per mille, ppm and percentage points.
- The concepts of rate of change and index, as well as methods for calculating interest and amortisation for different types of loans.
- The concept of a function, domain and range of a definition, and also properties of linear functions, and exponential functions.
- Representations of functions, such as in the form of words, shapes, functional expressions, tables and graphs.

- Differences between the concepts of equation, algebraic expressions and functions.

Probability and statistics

- Examination of how statistical methods and results are used in society and in science.
- The concepts of dependent and independent events, as well as methods for calculating probabilities in multi-stage random trials, using examples from games, and risk and safety assessments.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- Mathematical problems relevant to personal finances, societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **with elements of** mathematical symbols and other representations.
By giving examples students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve complex mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with some adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in symbolic algebra**. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing,** applying

and adapting mathematical models. Students can in a **balanced** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning, and evaluate this using **balanced** assessment **and further develop** their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with good adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT01c

Mathematics 1c

The course mathematics 1c covers points 1–7 under the heading Aim of the subject. The course covers basic knowledge in the subject.

Core content

Teaching in the course should cover the following core content:

Understanding of numbers, arithmetic and algebra

- Properties of a range of whole numbers, different number bases, and the concepts of prime numbers and divisibility.
- Methods of calculating in everyday life and for subjects typical of programmes, real numbers written in different forms, including powers with real exponents, and strategies for using digital tools.
- Generalisation of the rules of arithmetic to handle algebraic expressions.
- The concept of linear inequality.
- Algebraic and graphical methods for solving linear equations and inequalities, and exponential equations.

Geometry

- The concepts of sine, cosine and tangent, and methods of calculating angles and lengths of right angled triangles.
- The concept of vector and its representations, such as direction, length and points in a coordinate system.
- Addition and subtraction with vectors and scalar multiplication to produce a vector.
- Mathematical reasoning using basic logic, including implication and equivalence, and comparisons with how to argue in everyday contexts and in science subjects.
- Illustration of the concepts of definition, theorem and proof, such as the Pythagorean theorem and the sum of the angles of a triangle.

Relationships and change

- Advanced percentage concepts: per mille, ppm and percentage points.
- The concepts of rate of change and index, as well as methods for calculating interest and amortisation for different types of loans.
- The concept of a function, domain and range of a definition, and also properties of linear functions, and exponential functions.

- Representations of functions in the form of words, functional expressions, tables and graphs.
- Differences between the concepts of equation, inequality, algebraic expression and function.

Probability and statistics

- Examination of how statistical methods and results are used in society and in science.
- The concepts of dependent and independent events, as well as methods for calculating probabilities in multi-stage random trials, using examples from games, and risk and safety assessments.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- Mathematical problems relevant to personal finances, societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations.
Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **with elements of** mathematical symbols and other representations.
By giving examples students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty use** concepts and relationships between concepts to solve complex mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with some adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in symbolic algebra**. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing,** applying

and adapting mathematical models. Students can in a **balanced** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning, evaluate in **balanced** assessments **and further develop** their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with good adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT02a

Mathematics 2a

The course mathematics 2a covers points 1–7 under the heading Aim of the subject.

Core content

Teaching in the course should cover the following core content:

Understanding of numbers, arithmetic and algebra

- Methods for calculating budgets.
- Methods of calculating with powers with rational exponents.
- Strategies for formulating algebraic expressions, formulae and equations related to concrete situations and subjects typical of programmes.
- Handling the rules for squaring and factorising when solving equations.
- Linear equations and how analytical geometry links together geometric and algebraic expressions.
- Use of linear equations in problem solving situations.
- Algebraic and graphical methods for solving exponential and second-degree equations, and also linear equation systems.
- Solution of exponential equations through testing and graphical methods.

Geometry

- In-depth study of geometrical concepts chosen on the basis of the needs of subject typical of programmes, such as sine, cosine, tangents, vectors and symmetries.
- Mathematical reasoning using basic logic, including implication and equivalence and equivalence, and comparisons with how to reason in everyday and professional contexts.

Relationships and change

- The concept of function, domain and range of a definition. Applications of and properties of linear functions, and power, second-degree and exponential functions.
- Representations of functions, such as in the form of words, shapes, functional expressions, tables and graphs.
- Construction of graphs for functions and determining a function's value and setting it to zero, with and without digital tools.
- Differences between the concepts of equation, algebraic expressions and functions.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- How mathematics can be used as a tool in dealing with wide-ranging problem situations in subjects typical of a programme. The opportunities and limitations of mathematics in these situations.
- Mathematical problems of importance in societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations.
Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **with elements of** mathematical symbols and other representations.
By giving examples students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, stu-

dents switch **with some certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with some adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in symbolic algebra**. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing,** applying **and adapting** mathematical models. Students can in a **balanced** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning, evaluate in **balanced** assessments **and further develop** their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with certainty** in speech, written forms and

in action **and use** mathematical symbols and other representations **with good adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT02b

Mathematics 2b

The course mathematics 2b covers points 1–7 under the heading Aim of the subject.

Core content

Teaching in the course should cover the following core content:

Understanding of numbers, arithmetic and algebra

- Methods of calculating with powers with rational exponents.
- The concept of logarithms in solving exponential functions.
- Methods for calculating budgets.
- Linear equations and how analytical geometry links together geometric and algebraic expressions.
- The concept of linear equations.
- Handling the rules for squaring and factorising when solving equations.
- Extension of the number area through the introduction of the concept of complex numbers in connection with solving second-degree equations.
- Algebraic and graphical methods for solving exponential and second-degree equations, and also linear equation systems.

Geometry

- Use of fundamental classical theorems in geometry concerning similarity, congruence, and angles.

Relationships and change

- Properties of quadratic functions.
- Construction of graphs for functions and determining a function's value and setting it to zero, with and without digital tools.

Probability and statistics

- Statistical methods for reporting observations and data from surveys, including regression analysis.
- Orientation and discussion of correlation and causality.
- Methods for calculating different measures of central tendency and measures of dispersion including standard deviation.
- Properties of normally distributed material.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- Mathematical problems of importance in societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **with elements of** mathematical symbols and other representations.
By giving examples students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme.

In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with some adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme.
In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in symbolic algebra**. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing,** applying **and adapting** mathematical models. Students can in a **balanced** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning, evaluate in **balanced** assessments **and further develop** their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with good adaptation to purpose and situation**.

By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT02c

Mathematics 2c

The course mathematics 2 covers points 1–7 under the heading Aim of the subject.

Core content

Teaching in the course should cover the following core content:

Understanding of numbers, arithmetic and algebra

- The concept of logarithms, reasoning and applying logarithmic laws.
- Reasoning and handling algebraic identities including rules on quadratics and conjugates.
- The concept of linear equation systems.
- Algebraic and graphical methods for solving exponential, second degree and root equations, and linear equation systems with two or three unknown numbers.
- Extension of the number system through the introduction of the concept of complex numbers in connection with solving second-degree equations.

Geometry

- The concept of curves, straight lines and parabolic equations and how analytic geometry relates geometric and algebraic concepts.
- Use of fundamental classical theorems in geometry concerning similarity, congruence, and angles.

Relationships and change

- Properties of quadratic functions.
- Construction of graphs for functions and determining a function's value and setting it to zero, with and without digital tools.

Probability and statistics

- Statistical methods for reporting observations and data from surveys, including regression analysis.
- Methods for calculating different measures of central tendency and measures of dispersion including standard deviation.
- Properties of normally distributed material.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.

- Mathematical problems of importance in societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, written forms and in action **with elements of** mathematical symbols and other representations.
By giving examples students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work stu-

dents re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with some certainty** in speech, writing and in action **and use** mathematical symbols and other representations **with some adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in symbolic algebra**. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing,** applying **and adapting** mathematical models. Students can in a **balanced** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning, evaluate in **balanced** assessments **and further develop** their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students can express themselves **with certainty** in speech, written forms and in action **and use** mathematical symbols and other representations **with good adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT03b

Mathematics 3b

The course mathematics 3b covers points 1–7 under the heading Aim of the subject.

Core content

Teaching in the course should cover the following core content:

Algebra

- The concepts of polynomial and rational expressions, and generalization of the laws of arithmetic for dealing with these concepts.
- Algebraic and graphical methods for solving polynomial equations of higher degrees.

Relationships and change

- Use of the concept of geometric sums and linear optimisation in applications of relevance for subjects typical of programmes.
- Orientation to continuous and discrete functions, as well as the concept of limits.
- Properties of polynomial functions of higher orders.
- The concepts of secant, tangent, rate of change and derivatives of a function.
- Derivation and use of the rules of derivation for power and exponential functions, and also sums of functions.
- Introduction of the number "e" and its properties.
- Algebraic and graphical methods for determining the value of the derivative of a function.
- Algebraic and graphical methods for solving extreme value problems using sign tables and second derivatives.
- Relationship between the graph of a function and the first and second derivatives of a function.
- The concept of antiderivatives and definite integrals and the relationship between integrals and derivatives.
- Determining simple integrals in applications relevant for subjects typical of programmes.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.

- Mathematical problems of importance in societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students express themselves **with some certainty** in speech and writing **with elements of** mathematical symbols and other representations.
By giving examples students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty**, both with and without digital tools.

Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. **Furthermore, students can carry out simple mathematical proofs.** In addition, students express themselves **with some certainty** in speech and writing **and use** mathematical symbols and other forms of representation **with some adaptation to purpose and situation.**
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **define and in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
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By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural

history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT03c

Mathematics 3c

The course mathematics 3 covers points 1–7 under the heading Aim of the subject. The course covers advanced knowledge in the subject.

Core content

Teaching in the course should cover the following core content:

Arithmetic, algebra and geometry

- The concept of absolute values.
- The concepts of polynomial and rational expressions, and generalization of the laws of arithmetic for dealing with these concepts.
- Properties of the equation of a circle and unit circle in defining trigonometric concepts.
- Proof and use of cosine, sine and area theorems for an arbitrary triangle.

Relationships and change

- Orientation to continuous and discrete functions, as well as the concept of limits.
- Properties of polynomial functions of higher orders.
- The concepts of secant, tangent, rate of change and derivatives of a function.
- Derivation and use of the rules of derivation for power and exponential functions, and also sums of functions.
- Introduction of the number "e" and its properties.
- Algebraic and graphical methods for determining the value of the derivative of a function.
- Algebraic and graphical methods for solving extreme value problems using sign tables and second derivatives.
- Relationship between the graph of a function and the first and second derivatives of a function.
- The concept of antiderivatives and definite integrals and the relationship between integrals and derivatives.
- Determining simple integrals in applications relevant for subjects typical of programmes.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.

- Mathematical problems of importance in societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
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Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty**, both with and without digital tools.

Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. **Furthermore, students can carry out simple mathematical proofs.** In addition, students express themselves **with some certainty** in speech and writing **and use** mathematical symbols and other forms of representation **with some adaptation to purpose and situation.**
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Grade B

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Grade A

Students can **define and in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
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history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT04

Mathematics 4

The course mathematics 4 covers points 1–7 under the heading Aim of the subject. The course covers advanced knowledge in the subject.

Core content

Teaching in the course should cover the following core content:

Arithmetic, algebra and geometry

- Methods of calculating complex numbers written in different forms including rectangular and polar forms.
- The complex number plane, representation of complex numbers as points and vectors.
- Conjugates and absolute amounts of a complex number.
- Use and proof of de Moivre's formula.
- Algebraic and graphical methods for solving simple polynomial equations with complex roots and real polynomial equations of higher degrees, also by means of the factor theorem.
- Handling trigonometric expressions, and proof and use of trigonometric formulae including the Pythagorean trigonometric identity and the addition formulae.
- Algebraic and graphical methods for solving trigonometric equations.
- Different methods of proof in mathematics, with examples from the areas of arithmetic, algebra or geometry.

Relationships and change

- Properties of trigonometric functions, logarithmic functions, compound functions and absolute amounts as functions.
- Drawing graphs and their related asymptotes.
- Derivation and use of the rules of derivation for trigonometric, logarithmic, exponential and compound functions, and also the product and quotients of functions.
- Algebraic and graphical methods for determining integrals with and without digital tools, including estimates of magnitudes and probability distribution.
- The concept of differential equations and their properties in simple applications that are relevant to subjects typical of programmes.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- Mathematical problems of importance in societal life and applications in other subjects.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a simple nature**. These problems involve **a few concepts** and require **simple** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by applying **given** mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies and methods.
Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students express themselves **with some certainty** in speech and writing **with elements of** mathematical symbols and other representations.
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Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme.

In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty**, both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. **Furthermore, students can carry out simple mathematical proofs.** In addition, students express themselves **with some certainty** in speech and writing **and use** mathematical symbols and other forms of representation **with some adaptation to purpose and situation.**
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **define and in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
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symbols and other forms of representation **with good adaptation to purpose and situation**.
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MATMAT05

Mathematics 5

The course mathematics 5 covers points 1–7 under the heading Aim of the subject. The course covers advanced knowledge in the subject.

Core content

Teaching in the course should cover the following core content:

Relationships and change

- Strategies to set up and interpret differential equations as models for real situations.
- Use and solution of differential equations with digital tools in various fields which are relevant to subjects typical of programmes.

Discrete mathematics

- The concept of number, operations on sets, set theory notation and Venn diagrams.
- The concept of congruence of integers and calculating congruence.
- The concept of permutations and combinations.
- Methods of calculating the number of combinations and permutations, and reasoning over the validity of methods.
- The concept of graphs, different types of graphs and their properties, and some well-known problems from graphical theory.
- The concept of recursion and sequence.
- Mathematical induction with concrete examples from e.g. the area of number theory.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- Wide ranging problem situations in subject typical of a programme which also deepen knowledge of integrals and derivatives. The opportunities and limitations of Mathematic in these situations.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations.
Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
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Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students express themselves **with some certainty** in speech and writing **with elements of** mathematical symbols and other representations.
By giving examples, students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty**, both with and without digital tools.
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Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. **Furthermore, students can carry out simple mathematical proofs.** In addition, students express themselves **with some certainty** in speech and writing **and use** mathematical symbols and other forms of representation **with some adaptation to purpose and situation.**
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **define and in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
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By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.

MATMAT00S

Mathematics – specialisation

The course mathematics – specialisation covers points 1–7 under the heading Aim of the subject. The course covers advanced knowledge in the subject.

Core content

Teaching in the course should cover the following core content:
Areas of mathematics

- Covering one or more comprehensive areas of mathematics, such as linear optimisation, game theory, logic, differential equations, probability theorems, linear algebra, and computational mathematics for finance and population.

Problem solving

- Strategies for mathematical problem solving including the use of digital media and tools.
- How mathematics can be used as a tool in dealing with wide-ranging problem situations in subjects typical of a programme. The opportunities and limitations of mathematics in these situations.
- Mathematical problems related to the cultural history of mathematics.

KNOWLEDGE REQUIREMENTS

Grade E

Students can **in basic terms** describe the meaning of key concepts using **some** representations and **in basic terms** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme **in familiar situations**. In their work students handle **some simple** procedures, and solve tasks of a standard nature **with some certainty,** both with and without digital tools.
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Students can apply **simple** mathematical reasoning and evaluate in **simple** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. In addition, students express themselves **with some**

certainty in speech and writing **with elements of** mathematical symbols and other representations.
By giving examples, students relate something in **the course content** to its importance in other subjects, professional, societal life, and to the cultural history of mathematics. In addition, students can apply **simple** reasoning to the relevance of the examples.

Grade D

Grade D means that the knowledge requirements for grade E and most of C are satisfied.

Grade C

Students can **in detail** describe the meaning of key concepts using **some** representations and **in detail** describe relationships between the concepts. In addition, students switch **with some certainty** between different representations. Students can **with some certainty** use concepts and relationships between concepts to solve mathematical problems and problem situations in subjects typical of a programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty**, both with and without digital tools.
Students can formulate, analyse and solve mathematical problems. These problems involve **several** concepts and require **advanced** interpretations. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing and** applying mathematical models. Students can in a **simple** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded** mathematical reasoning and evaluate in **balanced** assessments their own reasoning and that of others, and differentiate between guesses and well grounded statements. **Furthermore, students can carry out simple mathematical proofs.** In addition, students express themselves **with some certainty** in speech and writing **and use** mathematical symbols and other forms of representation **with some adaptation to purpose and situation.**
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded** reasoning to the relevance of the examples.

Grade B

Grade B means that the knowledge requirements for grade C and most of A are satisfied.

Grade A

Students can **define and in detail** describe the meaning of key concepts using **several** representations and **in detail** describe relationships between the concepts. In addition, students switch **with certainty** between different representations. Students can **with certainty** use concepts and relationships between concepts to solve **complex** mathematical problems and problem situations in subjects typical of a

programme. In their work students handle **several** procedures, **including advanced and algebraic expressions,** and solve tasks of a standard nature **with certainty and in an effective way,** both with and without digital tools.
Students can formulate, analyse and solve mathematical problems **of a complex nature**. These problems involve **several** concepts and require **advanced** interpretations. **In problem solving, students discover general relationships that are presented in symbolic algebra**. In their work students re-express and transform realistic problem situations into mathematical formulations by **choosing,** applying **and adapting** mathematical models. Students can in a **balanced** assessment evaluate the plausibility of their results, and also that of selected models, strategies, methods **and their alternatives**.
Students can apply **well grounded and balanced** mathematical reasoning, evaluate in **balanced** assessments **and further develop** their own reasoning and that of others, and differentiate between guesses and well grounded statements. **Furthermore, students can carry out mathematical proofs**. In addition, students can express themselves **with certainty** in speech and writing **and use** mathematical symbols and other forms of representation **with good adaptation to purpose and situation**.
By giving examples, students relate something in **some of the course's sub-areas** to its importance in other subjects, professional, societal life and to the cultural history of mathematics. In addition, students can apply **well grounded and balanced** reasoning to the relevance of the examples.