Debt and Deficits: Fiscal Analysis with Stationary Ratios

John Y. Campbell Can Gao Ian W. R. Martin*

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Abstract

We introduce a new measure of a government's fiscal position that exploits cointegrating relationships among fiscal variables and output. The measure is a loglinear combination of tax revenue, government spending and the market value of government debt that—unlike the debt-GDP ratio—appears stationary in the US and 15 other developed countries. A weak fiscal position must ultimately be resolved by low future returns on government debt or by fiscal adjustment, a combination of high tax growth and low spending growth. Empirically, we find that debt returns play a negligible role and that fiscal adjustment predominantly consists of changes in spending growth. We also study shocks to taxes and spending, finding negligible responses of debt returns to these shocks which are instead associated with subsequent mean-reversion in tax and spending growth.

*John Campbell: Harvard University and NBER, https://scholar.harvard.edu/campbell/home. Can Gao: University of St. Gallen and Swiss Finance Institute, https://sites.google.com/view/can-gao/home. Ian Martin: London School of Economics, https://personal.lse.ac.uk/martiniw.We acknowledge helpful comments from Henning Bohn, Bernard Dumas, Martin Ellison, Hanno Lustig, Greg Mankiw, Casey Mulligan, Emi Nakamura, Jon Steinsson, Andrew Scott, Tuomo Vuolteenaho, and seminar participants at Copenhagen Business School, Harvard, London School of Economics, NBER Summer Institute 2023, University of British Columbia, and Washington University, Macro Finance Society Oslo Meeting 2024, Collegio Carlo Alberto, University of Turin, ESCP Business School, University of St. Gallen.

If a government is in a weak fiscal position, then over the long run holders of government debt must earn low returns, or taxes must rise, or spending must fall; or some combination of all three possibilities must occur. As we will show, this follows essentially as a matter of accounting. But which channel is most important empirically?

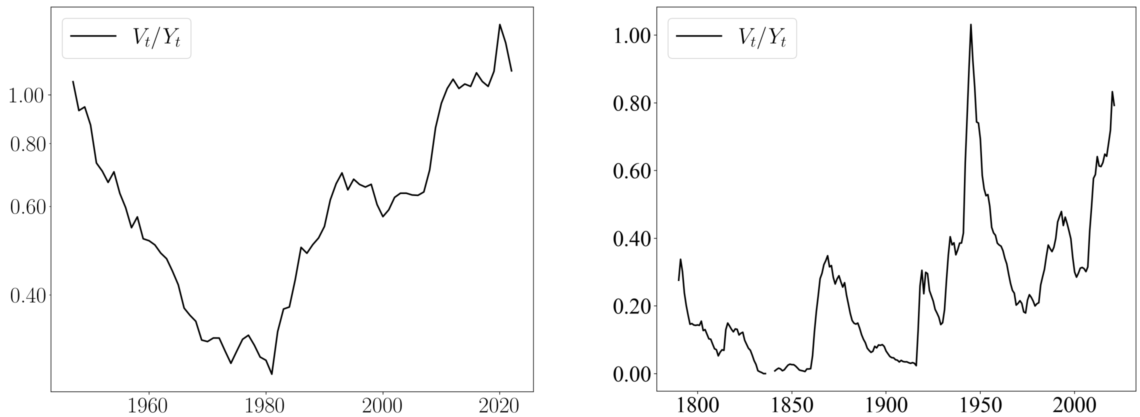
Any answer to this question requires a suitable definition of the "fiscal position." We will argue that some seemingly natural definitions are problematic. Certainly the primary surplus of a government is an essential ingredient. The primary surplus—the excess of tax revenue over government expenditure—is the flow of resources that the government devotes to servicing its debt. When it is positive, the growth rate of the value of the debt is less than the return on the debt. When it is negative—that is, when the government runs a primary deficit—the debt grows at a faster rate than the return on debt. Under the standard assumption that the expected return on the debt exceeds its growth rate, the value of the debt is the expected discounted value of the primary surpluses that will service it in the future.

To be useful in fiscal analysis, the primary surplus must be scaled in some way so that the resulting ratio is stationary. A common approach is to divide both the primary surplus and the value of debt by GDP to create the surplus-GDP and debt-GDP ratios. If either of these two ratios is stationary, the other should also be because of the present value relation that links surpluses and the value of debt. Many papers treat both ratios as stationary and ask what forces return the debt-GDP ratio to its unconditional mean (see, for example, Henning Bohn (1998, 1991, 2008), John H. Cochrane (2001, 2022, 2023), Olivier Blanchard (2019), and Zhengyang Jiang, Hanno Lustig, Stijn Van Nieuwerburgh and Mindy Z. Xiaolan (2021*b*)).

Contrary to this approach, we find that the debt-GDP ratio does not behave like a stationary time series in US data since World War II. As Figure 1, Panel a, shows, it has drifted persistently up and down for long periods of time. As one would expect, it shows no upward or downward trend; but it also shows no strong tendency to return to a constant mean. A unit root test fails to reject the null hypothesis that the debt-GDP ratio has a unit root, and cointegration tests fail to find statistically significant evidence that government debt is cointegrated with GDP. This nonstationarity helps to explain the (at first sight puzzling) finding in this literature that the debt-GDP ratio is not a successful predictor of fiscal outcomes.

From a theoretical perspective, the nonstationarity of debt-GDP is not particularly surprising: for example, Robert J. Barro (1979) writes, "There is no force that causes

Figure 1: The debt-GDP ratio is nonstationary in US data.



(a) Postwar sample, 1947–2022. Log scale. (b) Long sample, 1790–2022. Linear scale.

the ratio of debt to income to approach some target value".[1] Even if one believes that economic forces act to make the primary surplus-GDP ratio and the debt-GDP ratio truly stationary in the very long run—and the longer series shown in Figure 1, Panel b does not support this view—the persistence of these time series implies that it is inadvisable to model them using the standard techniques of stationary time-series analysis (John Y. Campbell and Pierre Perron, 1991).[2]

An alternative approach is to scale the primary surplus by the value of debt, and to work with the primary surplus-debt ratio. In an economy in which the return on the debt and the growth rate of the debt are stationary, the primary surplus-debt ratio should also be stationary.[3]

The primary surplus-debt ratio is analogous in the fiscal context to the dividend-price ratio on a stock. Just as a corporation pays dividends to the owners of its stock, so the government pays primary surpluses to the owners of its debt. This suggests the possibility of analyzing the primary surplus-debt ratio using a John Y. Campbell and Robert J. Shiller (1988) loglinearization to relate it to future log returns on debt and

[1]On the other hand, a *trend* in debt-GDP would be surprising as it would imply arbitrarily large or small values for this ratio in the distant future.

[2]Internet appendix IA.1 describes our data sources.

[3]Indeed, in postwar US data standard unit root tests reject the null hypothesis that the primary surplus-debt ratio has a unit root in favor of the alternative that it is stationary. However, this is also true of the primary surplus-GDP ratio. Primary surpluses are noisy enough that nonstationary dynamics in scaled surplus are hard for standard tests to detect. For this reason we do not emphasize unit root test results for ratios with the primary surplus in the numerator.

log growth rates of primary surpluses.

Two problems arise in doing so, and both result from the fact that the primary surplus can be negative. First, the log growth rate of the primary surplus is ill-defined when the surplus is negative. Second, an exogenous increase in the debt, which worsens the fiscal position of the government, can either raise or lower the primary surplus-debt ratio depending on whether the primary surplus is positive or negative. Thus, the effect of a given shock to the primary surplus-debt ratio depends on the sign of the ratio. Both these problems also afflict the standard analysis of the primary surplus-GDP ratio.

In this paper we develop an alternative loglinear analysis, related to the work of Chryssi Giannitsarou, Andrew Scott and Eric M. Leeper (2006) and Antje Berndt, Hanno Lustig and Şevin Yeltekin (2012), that solves these problems. Our approach is to approximate the primary surplus-debt ratio in a way that can be loglinearly related to the growth rates of tax revenue and of government expenditure. Both revenue and expenditure are always positive, so their log growth rates are well defined; and our loglinear approximation to the primary surplus-debt ratio has the appealing property that an increase in debt always reduces it, whether the primary surplus is currently positive or negative.

The approximations developed by Giannitsarou, Scott and Leeper (2006) and Berndt, Lustig and Yeltekin (2012) are similar in spirit but rely on the assumption that the tax revenue-debt and government expenditure-debt ratios are stationary, so that one can approximate around their means. In the US data we find to the contrary that neither of these ratios are stationary. Instead, their logs are cointegrated with a cointegrating vector that is close to but not equal to a unit vector. We use this finding of cointegration to develop an approximation, related to the work of Can Gao and Ian W. R. Martin (2021), that does not rely on inappropriate stationarity assumptions.

As the resulting measure of the fiscal position is stationary, it is a useful predictor variable for fiscal analysis. We use it to explore the dynamics of debt, tax revenue, and government expenditure in US data since World War II. We rely primarily on a vector autoregression (VAR) that includes the return on debt, the growth rate of tax revenue, the growth rate of output (which we include as a fundamental determinant of tax revenue and spending), and our measure of the fiscal position. The VAR system includes one extra lag of the fiscal position to ensure that the information set and hence our empirical results are identical (up to approximation error) whether we include tax revenue or spending growth in the VAR.

In the US, we find evidence for stationarity of the ratio of tax revenue to GDP, a result that contrasts with the nonstationarity of the debt-GDP ratio. Given this finding we also estimate a VAR model that includes the tax revenue-GDP ratio and explore the impact of this additional predictor variable.

Our main empirical findings are as follows. First, expected returns on government debt, while time-varying, are not variable or persistent enough to contribute importantly to the dynamics of the fiscal position. Instead, fiscal adjustment—changes in the growth rates of tax revenue and spending—accounts for the mean reversion of the fiscal position. Second, the primary driver of fiscal adjustment is the growth rate of spending rather than the growth rate of tax revenue. This result holds whether or not we include the tax revenue-GDP ratio in the VAR system. However it is particularly strong when that ratio is included, reflecting the fact that faster current growth of tax revenue raises the tax revenue-GDP ratio, predicting slower growth of GDP and eventually slower future growth in tax revenue. Third, the response of the fiscal position to shocks in tax revenue and government expenditures is determined almost entirely by mean-reversion in the growth rates of taxes and expenditures. Expected returns on government debt again have little importance, and the same is true for unexpected returns on debt contemporaneous with tax and expenditure shocks.

We repeat the analysis for 15 other developed countries: the UK, Canada, Japan, Switzerland, and 11 countries in the eurozone (Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, and Portugal). While the sample periods are shorter in these countries, reducing the power of unit root tests, the surplus-debt ratio appears stationary and the debt-GDP ratio nonstationary, as in the US. Unlike the US, we do not generally find that the tax revenue-GDP ratio is stationary so we avoid including this ratio in our non-US VAR systems. Our main findings hold up well across countries. Returns make only a minor contribution to the dynamics of the fiscal position, and in most countries (with the notable exception of Japan), fiscal adjustment is driven primarily by spending growth rather than by the growth rate of tax revenue.

Two caveats should be kept in mind when interpreting our results. First, because we conduct a reduced-form time-series analysis, we cannot make causal statements about fiscal dynamics. For example, our finding that an increase in the US tax revenue-GDP ratio predicts slower US GDP growth does not prove that high taxes cause lower growth as argued by Carmen M. Reinhart and Kenneth S. Rogoff (2010) and Alberto Alesina, Carlo Favero and Francesco Giavazzi (2020).

For the same reason we cannot resolve the debate about the fiscal theory of the price level (Thomas Sargent and Neil Wallace, 1981; Eric M. Leeper, 1991; Christopher A.

Sims, 1994; Michael Woodford, 1995; Cochrane, 2001, 2023). According to traditional analysis, the ability of the primary surplus-debt ratio to predict future fiscal adjustment is causal, in that a given value of the debt forces the government to run future primary surpluses that will pay it off. According to the fiscal theory of the price level, the predictive relationship reflects reverse causality: the debt has the value that is consistent with an exogenous path of future surpluses, as in a forward-looking asset pricing model of the sort analyzed by John Y. Campbell and Robert J. Shiller (1987); Campbell and Shiller (1988). If the debt promises to make fixed nominal payments, the required adjustment in value can occur largely through changes in the price level, although also in part through changes in long-term nominal interest rates (Cochrane, 2001). While we find that returns on government debt play a minor empirical role in adjustments to the fiscal position and to tax and spending shocks, advocates of the fiscal theory of the price level could argue that the developed countries we study happen to have exogenous data generating processes for primary surpluses that require only very modest variation in government debt returns.

Second, we take the returns on government debt as given, measuring them in the data without requiring them to satisfy the restrictions of any asset pricing model other than the weak restriction that they are high enough on average to rule out the existence of a bubble in government debt. We do not address the question, studied by Zhengyang Jiang, Hanno Lustig, Stijn Van Nieuwerburgh and Mindy Z. Xiaolan (2021*a*), of whether the measured return is too low to be consistent with the risk of the government debt, or the related question, discussed by Robin Greenwood, Samuel G. Hanson and Jeremy C. Stein (2015), Arvind Krishnamurthy and Annette Vissing-Jorgensen (2012), Ricardo Reis (2022), and Atif R. Mian, Ludwig Straub and Amir Sufi (2022), of whether government debt offers a convenience yield that investors value separately from its return.

The organization of the paper is as follows. In Section 1 we present a simple steady-state analysis of the primary surplus-debt ratio. This motivates the dynamic framework for fiscal analysis introduced in Section 2. We apply the framework empirically to US data in Sections 3 and 4, and to international data in Section 5. Section 6 concludes. An online appendix (John Y. Campbell, Can Gao and Ian W. R. Martin, 2024) presents supplementary details.

1 The primary surplus-debt ratio in steady state

By definition, the gross return on government debt is

$$R_{t+1} = \frac{V_{t+1} + T_{t+1} - X_{t+1}}{V_t}\,. \tag{1}$$

Here R_{t+1} is the return on debt from time t to $t+1$, V_t is the total market value of the debt in period t, T_{t+1} is tax income and X_{t+1} is expenditure. All variables are defined in real terms.

We define the primary surplus as $S_t = T_t - X_t$ and assume throughout that the gross return R_{t+1} is strictly positive: this rules out the possibility of a total default on all government debt obligations with zero recovery. Note that the debt return R_{t+1} should only be interpreted as a riskless interest rate in the special case in which all government debt is short-term real debt. We allow debt to be risky: the realized return on debt is low if, for example, real yields rise, or if there is a sudden unexpected inflation or explicit default.

As a first step toward a simple benchmark, let us imagine that conditional expectations of growth in tax, spending, and the debt are all equal to some constant, G.[4] Similarly, let us suppose that the conditionally expected return on debt equals R. Equation (1) then implies

$$R = \mathbb{E}_t \frac{V_{t+1}}{V_t} + \mathbb{E}_t \frac{T_{t+1}}{T_t}\frac{T_t}{V_t} - \mathbb{E}_t \frac{X_{t+1}}{X_t}\frac{X_t}{V_t} = G\left(1 + \frac{S_t}{V_t}\right)\,. \tag{2}$$

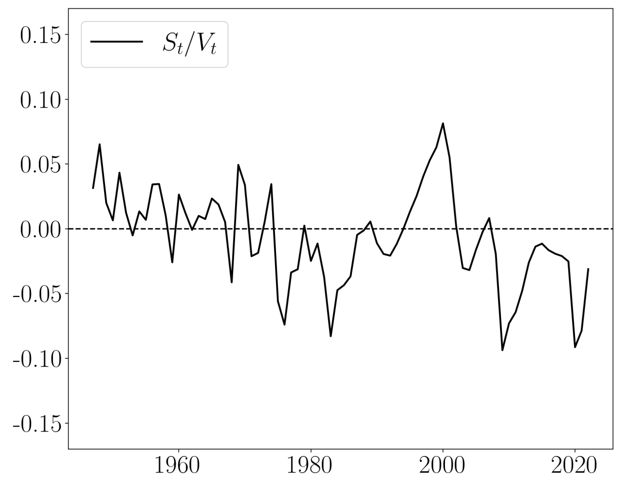
It follows that the primary surplus-debt ratio is a constant:

$$\log\left(1 + \frac{S_t}{V_t}\right) = \log R - \log G\,. \tag{3}$$

We write the ratio in this form for comparability with the more general analysis below. When $R > G$, the government must run primary surpluses to pay off its debt. By contrast, if $R \leq G$ the government need not run surpluses: even an unexpected increase in debt—for example, to fight a war—never needs to be paid off. In this case, the value

[4]This assumption is not unreasonable for unconditional expectations. Table IA.17 shows that the sample averages of log tax growth, log spending growth, and log debt growth are all approximately equal in our sample period. They are also all approximately equal to log GDP growth, consistent with the absence of a trend in the log debt-GDP ratio. Of course conditional expectations vary in the data, as we discuss later.

Figure 2: The surplus-debt ratio is stationary in postwar US data.



of the debt reflects the presence of a rational bubble. In our more general analysis of Section 2, we will rule out this possibility a priori.

Equation (3) exhibits the primary surplus-debt ratio as a natural quantity of interest, analogous to the dividend-price ratio in the Gordon growth model. Figure 2 shows the evolution of the surplus-debt ratio, S_t/V_t, in the US from 1947 to 2022. As the surplus can take negative values, we plot the series on a linear scale. (We provide a detailed description of our data sources in section IA.1 of the online appendix, and summary statistics are provided in appendix Table IA.17.) Although the surplus-debt ratio is not constant as it would be in a Gordon-growth-type model, it does appear to be stationary.[5]

2 A framework for fiscal analysis

The simple benchmark (3) is unrealistic in various important ways: for one thing, it implies that surplus cannot switch sign. To set up an empirically useful framework, we will have to account for the fact that the conditional expectations of returns and of the growth rates of tax revenue, spending, and debt all vary over time. We now present a general approach to doing so. Our framework does not restrict the time-series behavior

[5] This impression is supported by an augmented Dickey–Fuller (ADF) test, reported in Table IA.9, which rejects the presence of a unit root at the 99% confidence level. Although unit root tests can have poor finite-sample properties for ratios with noisy numerators such as the primary surplus, this finding, together with the theoretical presumption that the surplus-debt ratio should be stationary, gives us confidence to base our analysis on a stationarity assumption.

of conditional expectations, although the *unconditional* means of the growth rates of tax revenue, spending, and debt are all equal to each other (and to the unconditional mean of GDP growth) so that tax-debt, spending-debt, and debt-GDP ratios do not trend upwards or downwards over time.

To make a start, rewrite equation (1) as

$$R_{t+1} = \frac{V_{t+1}}{V_t}\left(1 + \frac{S_{t+1}}{V_{t+1}}\right). \tag{4}$$

Taking logs of (4), and using lower-case letters to denote logarithms of variables written with upper-case letters, we have

$$r_{t+1} = \Delta v_{t+1} + \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right). \tag{5}$$

An uncomfortable feature of the post-war data is that the time-series average of the surplus-debt ratio is negative over the sample period, as illustrated in Figure 2. If we believe that this sample average is an accurate measure of the true population average, then it follows from identity (5) that

$$\underbrace{\mathbb{E}\, r_{t+1}}_{\text{“}R\text{”}} - \underbrace{\mathbb{E}\, \Delta v_{t+1}}_{\text{“}G\text{”}} = \mathbb{E}\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) < 0\,. \tag{6}$$

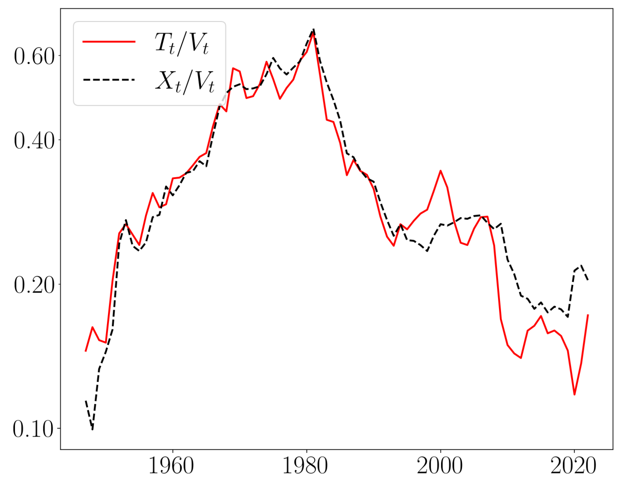
This is an “$R < G$” condition. But, as we will show, if the expected log return on the debt is less than its expected log growth rate, then we are forced to conclude that the value of the debt reflects the presence of a rational bubble. We rule out this possibility by imposing a positive population mean $\mathbb{E}\log(1 + S_t/V_t) > 0$ in our empirical work.

Figure 3 breaks the primary surplus $S_t = T_t - X_t$ into its constituent parts, plotting the tax-debt and spending-debt ratios separately. Again, the impression which emerges from the figure is confirmed by ADF tests: neither $\tau v_t = \log T_t/V_t$ nor $xv_t = \log X_t/V_t$ is stationary, despite the fact that the surplus-debt ratio is stationary. These facts place important constraints on how we set up our analysis.

2.1 A loglinear measure of the fiscal position

The measure of the surplus-debt ratio that appears on the right-hand side of (5) is similar to the dividend-price ratio measure, $\log(1 + D_{t+1}/P_{t+1})$, used by Gao and Martin (2021). It allows surplus to go negative; moreover, the measure is in natural units, in

Figure 3: The spending-debt and tax-debt ratios are nonstationary in postwar US data.



the sense that $\log(1 + S_{t+1}/V_{t+1})$ is approximately equal to S_{t+1}/V_{t+1} if surplus-debt is small. It can be written in terms of the log tax-debt ratio, $\tau v_t = \log(T_t/V_t)$, and the log spending-debt ratio, $xv_t = \log(X_t/V_t)$, as

$$\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = \log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right) . \tag{7}$$

To construct a tractable measure of the fiscal position, we linearize equation (7) in τv_t and xv_t. In doing so, we exploit the fact that while neither tax-debt, τv_t, nor spending-debt, xv_t, is stationary over the postwar sample, as discussed in the previous section and shown in Figure 3, they do appear to be cointegrated. Johansen tests (in both the trace and eigenvalue form) reject the null hypothesis that there is no cointegrating relationship between τv_t and xv_t at the 99% level: we can therefore treat $\tau v_t - \beta\, xv_t$ as stationary for some constant β. The cointegrating coefficient β will play an important role in our analysis. Likewise, $\log(1 + S_t/V_t)$ is stationary, as discussed in the previous section. We use these facts to guide our linearization.

Specifically, linearizing $\log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right)$ around $(\tau v_{t+1}, xv_{t+1}) = (\log a, \log b)$, where a and b are both positive, we have

$$\log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right) = k + \frac{1}{1 + a - b}\left(a\, \tau v_{t+1} - b\, xv_{t+1}\right) \tag{8}$$

up to higher order terms in τv_{t+1} and xv_{t+1}, where

$$k = \log(1 + a - b) + \frac{b \log b - a \log a}{1 + a - b}. \tag{9}$$

We choose a and b to satisfy two conditions. First, we want to linearize around the unconditional mean of $\log(1 + S_{t+1}/V_{t+1})$: that is, we require

$$\log(1 + a - b) = \mathbb{E} \log\left(1 + \frac{S_t}{V_t}\right). \tag{10}$$

As noted in the discussion following equation (6), we assume that $\mathbb{E}\log(1 + S_t/V_t) > 0$, or equivalently that $a > b$. This is equivalent to imposing an a priori constraint that the government must ultimately pay off its debt.

We write

$$\mathbb{E}\log(1 + S_t/V_t) = -\log \rho\,, \tag{11}$$

where the assumption that $\mathbb{E}\log(1 + S_t/V_t) > 0$ implies that the parameter ρ must lie between zero and one. In this notation, equation (10) becomes

$$1 + a - b = \frac{1}{\rho}. \tag{12}$$

Second, we want the right-hand side of (8) to be stationary, as the left-hand side is. Given the cointegrating relationship between τv_t and xv_t, this requires that

$$\frac{b}{a} = \beta\,, \tag{13}$$

where β is the cointegrating coefficient such that $\tau v_t - \beta\, xv_t$ is stationary. Since we have already assumed that $a > b$, the parameter $\beta < 1$. The parameters ρ and β both approach one in the limiting case where a approaches b.

Equations (12) and (13) jointly determine a and b in terms of β and ρ. We have

$$a = \frac{1}{1-\beta}\frac{1-\rho}{\rho} \quad \text{and} \quad b = \frac{\beta}{1-\beta}\frac{1-\rho}{\rho}. \tag{14}$$

In our empirical analysis of US data, we set $\rho = 0.999$ and $\beta = 0.995$. Equation (14) tells us that these choices correspond to $a = 0.200$ and $b = 0.199$. a and b are close to one another because ρ and β are close to one.

Plugging the expressions for a and b back into (8), we have our linearization

$$\log\left(1+\frac{S_{t+1}}{V_{t+1}}\right)=\log\left(1+e^{\tau v_{t+1}}-e^{x v_{t+1}}\right)=\underbrace{k+\frac{1-\rho}{1-\beta}\left(\tau v_{t+1}-\beta\, x v_{t+1}\right)}_{s v_{t+1}}, \tag{15}$$

where the first equality follows from the definition of surplus. Here k is as in equation (9) with a and b given by (14).

We will refer to the quantity on the far right-hand side of equation (15) as sv_{t+1} and will use it as our measure of the government's fiscal position. That is, we define

$$sv_t = k+\frac{1-\rho}{1-\beta}\left(\tau v_t-\beta\, x v_t\right) \tag{16}$$

where

$$k=\rho\log\rho+(1-\rho)\log\frac{1-\rho}{1-\beta}-\frac{1-\rho}{1-\beta}\beta\log\beta\,, \tag{17}$$

so that sv_t is a linearization of $\log(1+S_t/V_t)$ that, like $\log(1+S_t/V_t)$, is stationary.

The two quantities differ in one important way, however. As the level of debt rises with surplus held fixed, sv_t declines whether the surplus is positive or negative. This follows from the definition (16), given that ρ and β lie between zero and one. Combining this property with the standard positive response of sv_t to tax revenue and negative response to spending, we can think of sv_t as a measure of the fiscal position: it is high when the government is in a strong fiscal position, and low when the government is in a weak fiscal position. By contrast, the more conventional measures S_t/V_t and $\log(1+S_t/V_t)$ are harder to interpret: as the debt grows, they go *down* if the primary surplus is positive, but *up* if the surplus is negative.

2.2 A present value model for the fiscal position

The linearity of sv_t allows us to relate it to fundamentals in a linear present value framework. Inserting the linearization (15) into the exact identity (5), we have

$$r_{t+1}=\Delta v_{t+1}+sv_{t+1}\,. \tag{18}$$

Taking differences of (16) and rearranging, we have

$$(1-\rho)\Delta v_{t+1}=\frac{1-\rho}{1-\beta}\Delta\tau_{t+1}-\beta\frac{1-\rho}{1-\beta}\Delta x_{t+1}-\Delta sv_{t+1}\,. \tag{19}$$

We use (19) to eliminate Δv_{t+1} from (18), giving, after some rearrangement,

$$sv_t = (1-\rho)\left[r_{t+1} - \frac{1}{1-\beta}\Delta\tau_{t+1} + \frac{\beta}{1-\beta}\Delta x_{t+1}\right] + \rho\, sv_{t+1}\,. \tag{20}$$

We now solve forward in the usual way, to find that

$$sv_t = (1-\rho)\sum_{j=0}^{T-1}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right] + \rho^T sv_{t+T}\,. \tag{21}$$

Stationarity implies that sv_t is not explosive, so that $\lim_{T\to\infty}\rho^T sv_{t+T} = 0$. In the limit as $T\to\infty$, we therefore have the dynamic generalization of the static present value formula (3) that we were seeking:

$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right]. \tag{22}$$

In words, if the government is in a strong fiscal position (sv_t is high), then either the holders of government debt will earn high log returns, or taxes will grow slowly, or government expenditure will grow rapidly, or some combination of the above will occur, at some point in the future. This relationship is a loglinear approximation to an accounting identity, so it holds ex post. It also holds ex ante for rational expectations, and indeed for any subjective expectations that respect identities.

Four further points about equation (22) are worth noting. First, the right-hand side of (22) can be interpreted as a weighted average because $(1-\rho)\sum_{j=0}^{\infty}\rho^j = 1$. This means that we have the unconditional relationship

$$\mathbb{E}\, sv_t = \mathbb{E}\, r_t - \frac{1}{1-\beta}\,\mathbb{E}\,\Delta\tau_t + \frac{\beta}{1-\beta}\,\mathbb{E}\,\Delta x_t\,. \tag{23}$$

As noted at the beginning of Section 2, we must have equal unconditional growth rates of tax, spending, and debt so that fiscal ratios do not trend upwards or downwards over time. (This is borne out in postwar US data: log tax growth, log spending growth, and log debt growth have means of 0.031, 0.030, and 0.030, respectively.) Writing $\mathbb{E}\,\Delta\tau_t = \mathbb{E}\,\Delta x_t = \mathbb{E}\,\Delta v_t = g$, equations (18) and (23) each imply the relationship

$$\mathbb{E}\, sv_t = \mathbb{E}\, r_t - g\,, \tag{24}$$

analogous to an unconditional Gordon growth model.

Second, the discounting with discount factor $\rho < 1$ implies that the longer the various sources of fiscal adjustment are delayed, the larger they must ultimately be. This effect is stronger when ρ is low, as will be the case when returns on government debt are high relative to growth. In US data, however, returns are low relative to growth so we set $\rho = 0.999$ implying that this discounting effect is very weak.

Third, the multiplication of tax growth by $1/(1-\beta)$ and of spending growth by $\beta/(1-\beta)$—both of which are large numbers given that β is close to one—reflects the fact that when the average primary surplus is small relative to the average levels of tax revenue and government expenditure, small percentage changes in either taxes or spending have large proportional effects on the primary surplus and hence on our measure of the fiscal position. In our US analysis, we set $\beta = 0.995$ implying that the tax growth coefficient $1/(1-\beta) = 200$ and the spending growth coefficient $\beta/(1-\beta) = 199$. The slightly smaller coefficient for spending growth than for tax growth reflects the fact that when $\rho < 1$, the ratio of the level of spending to the level of tax revenue must be slightly less than one on average in order to pay off outstanding debt. Thus equal growth rates of the two variables have a smaller dollar impact for spending than for taxes. However with $\rho = 0.999$ the difference is very small.

Another way to understand this point is to use equation (14) to express β in terms of ρ and the loglinearization parameters a and b. We could express β either as a function of ρ and a or as a function of ρ and b; to keep things symmetrical, we do both and then average the resulting identities. This allows us to rewrite (22) as

$$sv_t = \sum_{j=0}^{\infty} \rho^j \left[(1-\rho) \left(r_{t+1+j} - \frac{\Delta\tau_{t+1+j} + \Delta x_{t+1+j}}{2} \right) + \rho\phi \left(\Delta x_{t+1+j} - \Delta\tau_{t+1+j} \right) \right], \quad (25)$$

where $\phi = (a+b)/2$. In our US analysis, $\phi = 0.1995$. Writing the identity in this way allows us to emphasize two conceptually distinct factors which matter for the interplay between debt and deficits. The first is captured by the parameter ρ, which one can think of as measuring the burden of debt: it is linked to the average size of the surplus that is required to service the debt, as discussed in the steady-state example of Section 1. When ρ is low, a large surplus is required to service each dollar of market value of the debt; at the other end of the spectrum, when $\rho = 1$ there is no debt burden at all because the debt need never be paid off. The second, captured by ϕ, measures the scale of tax and of spending in gross terms: it captures the overall size of the government relative to the value of its debt. The first term on the right hand side of equation (25) corresponds to the standard Gordon growth model, where growth is measured using the average of tax and spending, and the second term captures the effect of changing the growth rate of spending relative to the growth rate of tax revenue. When the government is large, as captured by the parameter ϕ, small changes in the relative growth rates of spending and taxes can have a large impact on the fiscal position.

Finally, when we use sv_t as a forecasting variable with the property that

$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right], \tag{26}$$

as follows on taking conditional expectations of (22), we should bear in mind that it is expected *log* returns that matter.[6] As Gao and Martin (2021) note, we can write

$$\mathbb{E}_t\, r_{t+1+j} = \log \mathbb{E}_t\, R_{t+1+j} - \frac{1}{2}\,\mathrm{var}_t\, r_{t+1+j} - \sum_{n=3}^{\infty}\frac{\kappa_t^{(n)}(r_{t+1+j})}{n!}, \tag{27}$$

where $\kappa_t^{(n)}(r_{t+1+j})$ is the nth conditional cumulant of the log return. If debt returns are conditionally lognormal, then the higher cumulants $\kappa_t^{(n)}(r_{t+1+j})$ are zero for $n \geq 3$, but even in this case, low expected log returns—a potential resolution of a scenario in which fiscal health is poor, i.e. sv_t is low—may be consistent with *high* expected simple returns if returns are volatile (that is, the second cumulant is large); and the gap between the two may be wider still if log returns are right-skewed (so that the third cumulant is large) or fat-tailed (so that the fourth cumulant is large); and so on.

Some of our results below analyze the importance of tax and spending separately. We also find it useful to define a combination of the two that we call *fiscal adjustment*:

$$f_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}. \tag{28}$$

Fiscal adjustment is the change in the stationary linear combination of τ_{t+1} and x_{t+1} defined by the cointegrating coefficient β. As β tends to one, fiscal adjustment approaches

[6]Related, Narayana R. Kocherlakota (2023) shows, in models driven by a discrete-time time-homogeneous Markov process, that infinite debt rollover can be sustained if the yield on an infinitely long-term zero-coupon bond is sufficiently low. Ian W. R. Martin and Stephen A. Ross (2019) show, in the finite-state Markov chain setting, that the infinitely long yield equals the unconditional expected log return on the long bond; in this case infinite debt rollover is possible if the expected log return on debt is sufficiently low. For us, the relevant quantity is the expected log return on the debt as a whole, as the government does not in practice finance itself through long-horizon zero-coupon borrowing.

the growth rate of the logarithmic surplus measure $\Delta\tau x_{t+1}$. With this definition, the identity (21) becomes

$$sv_t = (1-\rho)\sum_{j=0}^{T-1}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}f_{t+1+j}\right] + \rho^T sv_{t+T}, \tag{29}$$

so that in the limit as T approaches infinity, the analog of identity (22) is

$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}f_{t+1+j}\right]. \tag{30}$$

This equation highlights the distinction between returns on the debt and fiscal adjustment of taxes and spending as responses to the government's fiscal position.

3 US debt and deficits since World War II

3.1 Data, unit root tests, and linearization parameters

To implement our approach to fiscal analysis, we begin by studying debt and deficits in the US since World War II, the series already illustrated in Figures 1, 2, 3. To measure tax revenue and spending of the US federal government, we use annual data on total receipts, outlays, and interest payments from 1947 available on the FRED website and reported by the Office of Management and Budget (OMB). We use total receipts as T_t, and the difference between total outlays and interest payments as X_t.[7] To measure GDP and inflation, we use National Income and Product Accounts (NIPA) data from the FRED website.

Our framework requires that we measure the market value of the government debt, not the more readily available face value of the debt. We use market value data provided by the Federal Reserve Bank of Dallas. To calculate real returns on the debt we apply the accounting identity (1) to the time series of debt, tax revenue, and spending and adjust for inflation. In section IA.3 of the internet appendix we confirm the plausibility

[7]Receipts include taxes and other collections from the public. For example, social security taxes are counted as taxes, and therefore social security benefit payments must be treated as outlays. See table 17.1 in this file for details. Outlays are payments that liquidate obligations. Details are given in the chapter on outlays in this file. The US federal government also collects income from the public through market-oriented activities. Collections from these activities are subtracted from gross outlays, rather than being added to taxes and other governmental receipts. See table 18.1 in this file for details.

of the implied return series by regressing it on contemporaneous variables that explain the returns on short-term and long-term government debt: the short-term realized real interest rate and the change in the long-term bond yield. These regressions have high explanatory power and coefficients with the right sign and strong statistical significance.[8]

In section IA.4 of the internet appendix we report unit root test statistics and sample autocorrelations for the major time series: government debt returns, the growth rates of tax revenue, spending, and output, the ratio of debt to GDP, the ratios of taxes and spending to output and debt, and finally our loglinear measure of the fiscal position. As we have already discussed, the results indicate that returns and the growth rates of tax, spending, and GDP are all stationary in postwar US data; debt-GDP, spending-GDP, tax-debt, and spending-debt are all nonstationary; and tax-GDP and the fiscal position are both stationary.

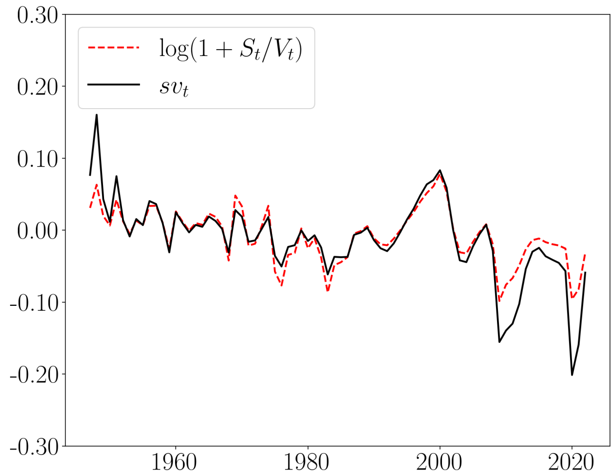
With these data in hand, the first task is to fix the linearization parameters ρ and β. As $\mathbb{E}\log(1+S_t/V_t) = -\log\rho$, we could in principle use the sample mean of $\log(1+S_t/V_t)$ to pin down ρ, given a sufficiently long sample. In postwar data, however, the average surplus-debt ratio is negative (see table IA.17 in appendix section IA.6), so this procedure would set ρ greater than one, and would bake in an "$R < G$" assumption. In order to impose a restriction that the government must pay off its debt, we therefore set ρ less than one as an a priori choice.

In our baseline analysis, we set $\rho = 0.999$ so that the implied unconditional expectation of $\log(1+S_t/V_t)$ is not too far from its sample mean in postwar data. For consistency with equation (24) and the surrounding discussion, we demean returns, tax growth and the fiscal position in our VAR estimation using theoretical restrictions. As the sample means of $\Delta\tau_t$, Δx_t, and Δv_t are 0.031, 0.030, and 0.029, respectively, we set g equal to 0.030 and impose $\mathbb{E}\,\Delta\tau_t = \mathbb{E}\,\Delta x_t = \mathbb{E}\,\Delta v_t = g$. We set $\mathbb{E}\,sv_t$ equal to $-\log\rho = 0.001$ and $\mathbb{E}\,r_t$ equal to $\mathbb{E}\,sv_t + g = 0.031$.

Finally, we choose β so that our measure of the fiscal position, sv_t, optimally approximates $\log(1 + S_t/V_t)$ in a least-squares sense. That is, β is chosen to solve the

[8] The R^2 for a univariate OLS regression of the implied bond return on the real short-term interest rate is 55% and the slope coefficient has a t statistic above 9; the R^2 for a univariate OLS regression on the change in the 10-year nominal bond yield is 61% and the slope coefficient has a t statistic above 8; and a multivariate regression including both explanatory variables has an R^2 of 85% and t statistics above 8 for both variables.

Figure 4: sv_t and $\log(1 + S_t/V_t)$ in postwar US data.



problem

$$\min_{\beta} \sum_t \left(\log\left(1 + S_t/V_t\right) - \underbrace{\left[k + \frac{1-\rho}{1-\beta} \left(\tau v_t - \beta\, x v_t\right) \right]}_{sv_t} \right)^2 , \tag{31}$$

where k is given in equation (17). With $\rho = 0.999$, this procedure sets $\beta = 0.995$. As required by our theory, both ρ and β lie between zero and one although they are close to one.

The time series of sv_t implied by these choices of ρ and β is shown in Figure 4, together with $\log(1 + S_t/V_t)$ which it approximates. Both sv_t and $\log(1 + S_t/V_t)$ are negative for extended periods of time, which is entirely consistent with our methodology. All our parameter choices have done is rule out the possibility that the means of these series are negative unconditionally, for all time. We conduct a sensitivity analysis in Section 3.6, where we show that plausible changes to the chosen value of ρ have little effect on our conclusions.

3.2 VAR estimation

The approximate identity (22) relates our measure of the fiscal position, sv_t, to future debt returns, tax growth, and spending growth. It formalizes the fact that when the government is in a weak fiscal position (i.e., sv_t is low) we must subsequently have some combination of low debt returns, high tax growth, and low spending growth.

To determine which of these channels is most important empirically, we estimate a VAR in the variables r_t, $\Delta\tau_t$, Δy_t, and sv_t. We include r_t, $\Delta\tau_t$, and sv_t for obvious reasons, given our interest in the identity (22). We include GDP growth, Δy_t, because of its importance for forecasting the other variables in the VAR: for example, we expect a larger economy to be able to raise a larger amount of tax revenue.

We do not include Δx_t as it is mechanically related to sv_t, sv_{t-1}, r_t and $\Delta\tau_t$ via the approximate identity (20). Indeed, we treat the identity as holding exactly, so that we can infer Δx_t using variables included in the VAR,

$$\frac{\beta}{1-\beta}\Delta x_t = \frac{sv_{t-1} - \rho sv_t}{1-\rho} - r_t + \frac{1}{1-\beta}\Delta\tau_t\,. \tag{32}$$

Note however that inferring Δx_t is possible only if we include an additional lag of the fiscal position, sv_{t-1} as well as sv_t, in the system. We include this additional lag so that (except for approximation error) our results are invariant to the decision to include $\Delta\tau$ in the VAR rather than Δx.[9]

The estimated VAR is shown in the first four columns of Table 1. The fiscal position sv_{t+1} is relatively predictable, with R^2 of almost 70%, and is strongly predicted by its lag. A strong fiscal position (high sv_t) forecasts high returns for debt holders and low tax growth. GDP growth (Δy_t) is a highly significant forecaster of tax growth with a coefficient above one, consistent with the presence of increasing marginal tax rates.

The last two columns of Table 1 show imputed coefficients for spending growth and fiscal adjustment $f_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}$. A strong fiscal position predicts high spending growth, but the effect operates with a lag. When the effects on tax and spending are combined in the fiscal adjustment measure, growth (high Δy_t) and a poor fiscal position (low sv_t and sv_{t-1}) forecast large fiscal adjustment.

[9] See Tom Engsted, Thomas Q. Pedersen and Carsten Tanggaard (2012) for a thoughtful discussion of this issue. Our approach can also be understood as an extension of the approach of John H. Cochrane (2008). Working on the topic of equity market predictability, Cochrane estimates a model whose only predictor variable is a valuation ratio analogous to our sv_t. He emphasizes the linkage between predictions of returns, cash flow growth, and future valuation ratios in that model. We extend his model by adding lagged dependent variables while continuing to include one additional lag of the valuation ratio (sv_t, in our context).

Table 1: VAR coefficient estimates. US data, 1947–2022.

OLS standard errors are reported in square brackets. The last two columns show the imputed coefficients for spending growth and $f_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}$.

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	sv_{t+1}	Δx_{t+1}	f_{t+1}
r_t	0.376	−0.21	0.046	−0.068	0.127	−0.337
	[0.115]	[0.114]	[0.053]	[0.047]	[0.205]	[0.245]
$\Delta\tau_t$	−0.049	0.019	−0.039	−0.077	0.405	−0.385
	[0.097]	[0.096]	[0.045]	[0.04]	[0.173]	[0.206]
Δy_t	0.209	1.703	0.191	0.354	−0.065	1.767
	[0.266]	[0.263]	[0.122]	[0.109]	[0.473]	[0.564]
sv_t	0.513	−0.763	−0.157	0.875	−0.135	−0.629
	[0.263]	[0.26]	[0.121]	[0.108]	[0.467]	[0.557]
sv_{t-1}	−0.12	0.118	0.207	−0.194	1.092	−0.968
	[0.275]	[0.272]	[0.126]	[0.113]	[0.487]	[0.582]
R^2	19.59%	48.52%	10.14%	69.59%	19.47%	31.18%

3.3 Decomposing the variance of the fiscal position

We can use the VAR to understand what fluctuations in the fiscal position, sv_t, imply about the subsequent evolution of debt returns, tax growth, and spending growth. Stacking the variables into a vector $\boldsymbol{z}_{t+1} = (r_{t+1}, \Delta\tau_{t+1}, \Delta y_{t+1}, sv_{t+1}, sv_t)'$, we can arrange the entries of Table 1 into a coefficient matrix $\boldsymbol{A}$ such that $\mathbb{E}_t\, \boldsymbol{z}_{t+j} = \boldsymbol{A}^j \boldsymbol{z}_t$. If we write $\boldsymbol{e}_n$ for a vector with one in the n'th entry and zeroes elsewhere, we therefore have $\mathbb{E}_t\, r_{t+j} = \boldsymbol{e}_1' \boldsymbol{A}^j \boldsymbol{z}_t$, $\mathbb{E}_t\, \Delta\tau_{t+j} = \boldsymbol{e}_2' \boldsymbol{A}^j \boldsymbol{z}_t$, and so on.

We use the identity (29) to derive finite-horizon variance decompositions in the form

$$1 = \frac{\text{cov}(sv_t, (1-\rho)\sum_{j=0}^{T-1} \rho^j\, \mathbb{E}_t\, r_{t+1+j})}{\text{var } sv_t} + \frac{\text{cov}(sv_t, -(1-\rho)\sum_{j=0}^{T-1} \rho^j\, \mathbb{E}_t\, \frac{1}{1-\beta} f_{t+1+j})}{\text{var } sv_t} + \\ + \frac{\text{cov}(sv_t, \rho^T\, \mathbb{E}_t\, sv_{t+T})}{\text{var } sv_t}. \tag{33}$$

This decomposition can be derived by taking time-t conditional expectations of both sides of (29), computing covariances with sv_t and, finally, scaling by the variance of sv_t so that the three terms on the right-hand side of (33) add up to 100%. It allows us to formalize the statement with which we began: given that the fiscal position varies, it must, for any given horizon T, forecast some combination of future returns on the debt,

Table 2: A variance decomposition for the fiscal position, sv_t, in postwar US data. All quantities are measured in percent. Bootstrapped 95% confidence intervals are reported in square brackets.

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	23.2	78.0	65.7
	$[0.0, 0.1]$	$[10.7, 36.3]$	$[65.0, 90.6]$	$[35.2, 89.1]$
3	0.1	68.0	33.2	66.2
	$[0.0, 0.2]$	$[34.4, 98.3]$	$[2.9, 66.7]$	$[31.6, 95.9]$
10	0.2	99.1	2.0	74.6
	$[0.0, 0.5]$	$[71.9, 103.0]$	$[-1.9, 29.2]$	$[32.6, 117.7]$
∞	0.2	101.1	0.0	75.1
	$[0.0, 0.6]$	$[100.7, 101.3]$	$[-0.0, 0.0]$	$[31.1, 125.1]$

future fiscal adjustment, and/or persistent variation in the future fiscal position. As we let the horizon increase, the contribution of the future fiscal position declines to zero and we are left with a two-variable infinite-horizon variance decomposition for the fiscal position.

Table 2 reports the results of this exercise over various different horizons T. At each horizon, we report the three terms on the right-hand side of (33) in the columns labelled "return", "fiscal adjustment", and "future sv". These are measured in percent, and the three columns add up to approximately 100% at each horizon.[10]

Bootstrapped 95% confidence intervals for these estimates are shown in square brackets under the point estimates. Each bootstrap sample is computed by first drawing a new VAR coefficient matrix using the point estimates and the covariance matrix of the estimated coefficients. Using this VAR coefficient matrix, we generate the news series and do the variance decomposition. We repeat this procedure 2,000 times and report the 2.5% and 97.5% quantiles.

At short horizons, variation in sv_t is largely reflected in short-run future sv_t: if the fiscal position is weak this year, it probably will be next year too. But the component explained by future sv_t decays at long horizons, and reaches zero in the long run; and, at all horizons, there is essentially no relationship between the fiscal position and expected real returns. (This last fact contrasts with the evidence that dividend yields do forecast

[10] If the loglinear approximation were exact, the three columns would add up to exactly 100%.

returns on the stock market.)

As a result, the fiscal position sv_t must in the long run forecast fiscal adjustment. Specifically, we find that a poor fiscal position (low sv_t) is associated with high expected tax growth and/or low expected spending growth over the medium and long run.

As fiscal adjustment can be split into the contribution of tax increases and expenditure cuts, $f_{t+1+j} = \Delta\tau_{t+1+j} - \beta\Delta x_{t+1+j}$, the dominant second term in (33) can be decomposed further, as

$$\frac{\text{cov}(sv_t, -(1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}f_{t+1+j})}{\text{var}\,sv_t} = \frac{\text{cov}(sv_t, -(1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}\Delta\tau_{t+1+j})}{\text{var}\,sv_t} + \\ + \frac{\text{cov}(sv_t, (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{\beta}{1-\beta}\Delta x_{t+1+j})}{\text{var}\,sv_t}. \tag{34}$$

The fourth column of Table 2, labelled "spending ratio", reports the share of the contribution of fiscal adjustment that reflects adjustments in spending rather than tax: that is, it reports the ratio of the second term on the right-hand side of (34) to the term on the left-hand side. At medium and long horizons, the point estimates suggest that around three quarters of the variation in fiscal adjustment is adjustment in spending, as opposed to adjustment in taxes. The confidence intervals are fairly wide, however, and the lower ends of the confidence intervals have spending accounting for around one third of the variation in fiscal adjustment.

3.4 Implications of a stationary US tax-GDP ratio

Figure 5 plots US tax revenue-GDP and spending-GDP over time. By now it may come as no surprise that spending-GDP is not stationary. But the log tax-GDP ratio, $\tau y_t = \log T_t/Y_t$, does appear to be stationary in postwar US data. This pattern, confirmed by the unit root tests reported in the internet appendix section IA.4, supplies us with another stationary variable to take into account when we analyze fiscal dynamics.

Table 3 reports results for a VAR that includes τy_t, and so takes into account the stationary relationship between tax and output. The tax-GDP ratio is quite predictable, notably by its own lag and by tax growth, and in turn it predicts high returns on debt and low future tax growth.

We can use this VAR to conduct a variance decomposition analogous to the one reported in Table 2. The results are shown in Table 4. As before, we find that variation

Figure 5: The spending-GDP ratio is nonstationary, but the tax-GDP ratio is stationary in postwar US data. Log scale.

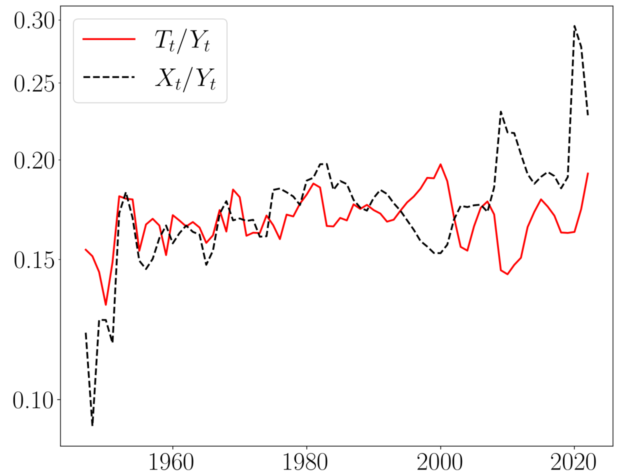


Table 3: VAR coefficient estimates for a system that includes the tax-GDP ratio, τy_t. US data, 1947–2022.

OLS standard errors are reported in square brackets. The last two columns show imputed coefficients for spending growth and for $f_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}$.

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	sv_{t+1}	τy_{t+1}	Δx_{t+1}	f_{t+1}
r_t	0.313	−0.112	0.074	−0.081	−0.186	0.294	−0.405
	[0.116]	[0.110]	[0.054]	[0.049]	[0.097]	[0.199]	[0.253]
$\Delta\tau_t$	−0.144	0.165	0.002	−0.097	0.163	0.655	−0.487
	[0.105]	[0.099]	[0.048]	[0.044]	[0.087]	[0.180]	[0.229]
Δy_t	0.414	1.384	0.103	0.398	1.281	−0.607	1.988
	[0.277]	[0.262]	[0.127]	[0.116]	[0.231]	[0.475]	[0.603]
sv_t	0.398	−0.584	−0.108	0.850	−0.477	0.169	−0.752
	[0.261]	[0.248]	[0.120]	[0.109]	[0.218]	[0.449]	[0.570]
sv_{t-1}	−0.118	0.115	0.206	−0.193	−0.092	1.087	−0.966
	[0.267]	[0.253]	[0.123]	[0.112]	[0.222]	[0.458]	[0.581]
τy_t	0.189	−0.292	−0.081	0.040	0.789	−0.498	0.203
	[0.090]	[0.085]	[0.042]	[0.038]	[0.075]	[0.155]	[0.196]
R^2	24.23%	55.23%	14.76%	70.27%	73.15%	29.97%	32.23%

Table 4: Variance decomposition of fiscal position sv_t, based on a VAR that includes the tax-GDP ratio, τy_t, in postwar US data.

All quantities are measured in percent. Bootstrapped 95% confidence intervals are reported in square brackets.

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	23.2	78.0	65.7
	$[0.0, 0.1]$	$[11.0, 36.7]$	$[64.6, 90.3]$	$[41.2, 85.6]$
3	0.1	70.3	31.0	74.1
	$[0.0, 0.2]$	$[38.3, 101.1]$	$[0.2, 63.0]$	$[54.2, 96.1]$
10	0.1	100.7	0.5	102.7
	$[-0.1, 0.3]$	$[77.7, 108.0]$	$[-6.8, 23.2]$	$[90.0, 130.1]$
∞	0.1	101.2	0.0	101.9
	$[-0.1, 0.4]$	$[100.9, 101.4]$	$[-0.0, 0.0]$	$[89.5, 135.3]$

in the government's fiscal position reflects expected future fiscal adjustment rather than expected future bond returns. What is new, relative to the earlier results, is that in this system fiscal adjustment takes place almost entirely through changes in expected spending growth as opposed to expected tax growth; and the confidence intervals for the contribution of spending are much smaller. This is the case because the fiscal position has little ability to forecast GDP growth (as shown in Table 3). It must therefore also have little ability to forecast tax growth, given the stationarity of the tax-GDP ratio.

3.5 Local projections

Our approximate identities (22) and (26) make no assumptions about the data-generating process. When we carry out variance decompositions, however, we are assuming that the VAR(1) system estimated in Table 1 accurately summarizes the data. Òscar Jordà (2005), Mikkel Plagborg-Møller and Christian K. Wolf (2021), and Dake Li, Mikkel Plagborg-Møller and Christian K. Wolf (2022) have argued for a local projection approach that imposes less structure on the underlying multivariate dynamic system.

Table 5 therefore reports results for an approach based on local projections at horizons of 1, 3, and 10 years, in the same format used in Tables 2 and 4 (except that Newey–West standard errors are reported in square brackets). Full details of our methodology are provided in Section IA.6.3 of the Appendix.

Table 5: Local projections, US data 1947-2022.

The table reports Newey–West standard errors with lags of 2, 5, and 15, respectively, at horizons $T = 1, 3$ and 10. The standard error for the spending ratio is computed by the delta method using Newey–West standard errors of $\beta_{\tau,T}$ and $\beta_{x,T}$.

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	26.2	73.8	67.5
	[0.0]	[7.4]	[7.4]	[20.8]
3	0.0	56.1	43.9	70.5
	[0.1]	[10.8]	[10.8]	[21.4]
10	–0.1	77.0	23.1	79.2
	[0.2]	[24.6]	[24.5]	[29.1]

3.6 The impact of the average surplus-debt ratio

Our analysis started from an assumption that the government debt does not have a bubble component. This implies that the unconditional average surplus-debt ratio $\mathbb{E}\log(1 + S_t/V_t) = -\log\rho$ must be positive. In our baseline analysis, we set $\rho = 0.999$ so that the implied unconditional expectation of $\log(1 + S_t/V_t)$ is not too far from its sample mean in postwar US data, but it is reasonable to ask how sensitive our results are to this assumption.

To understand the potential importance of the value of ρ, it is instructive to consider the limiting "$R = G$" case in which ρ and β both equal one and the unconditional expected return on debt equals the unconditional expected growth rate of the debt. In this case,

$$sv_t = \tau v_t - xv_t = \log\frac{T_t}{X_t} = \tau x_t. \tag{35}$$

The quantity τx_t can be interpreted as a logarithmic measure of the primary surplus. The level of the debt drops out in this case because debt can be rolled over forever and need never be paid off. Moreover, returns drop out of equation (22) in the limit: it simplifies to

$$sv_t = \tau x_t = \sum_{j=0}^{\infty} \left[-\Delta\tau_{t+1+j} + \Delta x_{t+1+j}\right]. \tag{36}$$

In this limiting case the future growth rates of tax revenue and spending are simply required to offset the current fiscal position (which equals the logarithmic surplus measure

τx_t in this case), so that primary surpluses are transitory rather than permanent.

Given this result, one might be concerned that our results follow mechanically from our choice of $\rho = 0.999$. However, in fact our major conclusions are not sensitive to the choice of ρ in a reasonable range. To demonstrate this, in appendix section IA.6.5 we reproduce the variance decompositions of Sections 3.3 and 4 for a range of values between $\rho = 0.999$ and $\rho = 0.75$. These different values of ρ represent different assumptions about the true unconditional population expectation, $\mathbb{E}\log(1+S_t/V_t)$, ranging from 0.1% when $\rho = 0.999$, through 4.1% when $\rho = 0.96$, to 28.8% when $\rho = 0.75$. Lower values of ρ are associated with higher values of $\mathbb{E}\log(1 + S_t/V_t)$; loosely speaking, lower ρ represents higher "$R - G$", so that issuing debt is more burdensome. The unconditional mean of the log return on government debt is 3.1% when $\rho = 0.999$, 7.1% when $\rho = 0.96$, and 31.8% when $\rho = 0.75$. We do not consider values of ρ below about 0.96 to be reasonable: we include them merely to show how our results would change in a world in which R is much higher than G.

As ρ influences the choice of β in problem (31) and the linearized variable sv_t in our VAR, we recalculate β and reestimate the VAR for each value of ρ. As in our baseline VAR, we impose consistency on our model by de-meaning with theoretical means, as discussed in Section 3.1. The appendix also presents tables showing the effect of varying β away from the estimated values, for ρ between 0.999 and 0.95.

In each table in appendix section IA.6.5 the first five columns report the various values of ρ together with the associated implied unconditional mean return on government debt, the estimated value of β, the approximation error in (31), and the maximum eigenvalue of the coefficient matrix which must be smaller than one in magnitude in order that the estimated system does not have explosive dynamics. The rightmost three columns report the variance decomposition at an infinite horizon, the share of variation in the fiscal position attributable to movements in returns, tax growth, and spending growth.

As ρ declines, both returns and taxes have a somewhat greater role to play. The increasing importance of returns with lower ρ is consistent with the fact that our VAR model predicts time-varying near-term returns but predicts almost constant returns in the distant future. The total weight on forecasts of all future returns is invariant to ρ in the identity (33), but as ρ declines the identity places relatively more weight on near-term forecasts which are those that vary over time.

Although the variance share of returns increases as ρ declines, the increase is modest. In a VAR that excludes the tax-income ratio τy_t, the variance share of returns is 0.2% at $\rho = 0.999$, 18.8% at $\rho = 0.96$, and 15.4% if we drive ρ down to the implausible value of

0.75. Our main conclusions—that variability in sv_t is predominantly resolved by fiscal adjustment, the bulk of which is driven by changes in spending—survive at all levels of ρ. These conclusions are also robust to variation in β and to the inclusion of the tax-income ratio τy_t in the VAR. Our variance decompositions for short-run tax and spending news are similarly robust to the values of ρ and β. Again returns become somewhat more important as ρ declines, but they never play more than a minor supporting role in the responses to tax and spending shocks.

4 Decomposing the responses to tax and expenditure shocks

As our framework allows us to analyze the behavior of tax and spending separately, we can also ask whether deficits driven by shocks to taxes look different from deficits driven by shocks to spending.

We address this question by using the identity (21) to explore the implications of unexpected shocks to taxes or spending. Applying the "news operator", $\Delta \mathbb{E}_{t+1} = \mathbb{E}_{t+1} - \mathbb{E}_t$, to both sides of (21) and rearranging, we have

$$\underbrace{\Delta \mathbb{E}_{t+1} \tau_{t+1}}_{\substack{\text{short-run tax news:} \\ N_{\text{SR tax},t+1}}} = (1-\beta) \underbrace{\Delta \mathbb{E}_{t+1} \sum_{j=0}^{T-1} \rho^j r_{t+1+j}}_{\text{return news: } N_{\text{return},t+1}} - \underbrace{\Delta \mathbb{E}_{t+1} \sum_{j=1}^{T-1} \rho^j \Delta\tau_{t+1+j}}_{\substack{\text{long-run tax news:} \\ N_{\text{LR tax},t+1}}} + \\ + \underbrace{\beta \, \Delta \mathbb{E}_{t+1} \sum_{j=0}^{T-1} \rho^j \Delta x_{t+1+j}}_{\text{spending news: } N_{\text{SR tax},t+1}} + \frac{1-\beta}{1-\rho} \underbrace{\Delta \mathbb{E}_{t+1} \rho^T sv_{t+T}}_{\substack{\text{future fiscal position news:} \\ N_{\text{future sv},t+1}}} . \tag{37}$$

This identity allows us to trace out the consequences of an unexpected shock to taxes. We refer to such a shock as short-run tax news, $N_{\text{SR tax},t+1} = \Delta \mathbb{E}_{t+1} \tau_{t+1}$. A positive short-run tax shock must be reflected in some combination of (i) news about returns, $N_{\text{return},t+1} = \Delta \mathbb{E}_{t+1} \sum_{j=0}^{T-1} \rho^j r_{t+1+j}$; (ii) news about declines in long-run tax growth, $N_{\text{LR tax},t+1} = \Delta \mathbb{E}_{t+1} \sum_{j=1}^{T-1} \rho^j \Delta\tau_{t+1+j}$; (iii) news about spending growth, $N_{\text{spending},t+1} = \Delta \mathbb{E}_{t+1} \sum_{j=0}^{T-1} \rho^j \Delta x_{t+1+j}$; and (iv) news about the future fiscal position, $N_{\text{future sv},t+1} = \Delta \mathbb{E}_{t+1} \rho^T sv_{t+T}$. This last term becomes negligible once the horizon, T, is sufficiently long.

Taking covariances of both sides of (37) with short-run tax news, $N_{\text{SR tax},t+1} =$

Table 6: A variance decomposition for short-run tax news in postwar US data.

horizon	return	fiscal adjustment	future sv	spending ratio
1	-0.1	-41.6	143.3	100.0
	$[-0.2, -0.1]$	$[-52.2, -31.0]$	$[132.7, 153.9]$	$[100.0, 100.0]$
3	0.0	13.2	88.3	-209.5
	$[-0.2, 0.2]$	$[-60.1, 75.3]$	$[26.2, 161.7]$	$[-1360.5, 1525.3]$
10	0.3	99.0	2.2	25.6
	$[0.0, 0.7]$	$[43.9, 127.0]$	$[-26.0, 57.1]$	$[-12.7, 61.7]$
∞	0.3	101.2	0.0	27.1
	$[-0.0, 0.9]$	$[100.6, 101.6]$	$[0.0, 0.0]$	$[4.0, 95.2]$

$\Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}$, and rearranging, we have

$$1 = \frac{\operatorname{cov}\left((1-\beta)N_{\text{return},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var} N_{\text{SR tax},t+1}} + \frac{\operatorname{cov}\left(-N_{\text{LR tax},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var} N_{\text{SR tax},t+1}}$$
$$+ \frac{\operatorname{cov}\left(\beta N_{\text{spending},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var} N_{\text{SR tax},t+1}} + \frac{\operatorname{cov}\left(\frac{1-\beta}{1-\rho} N_{\text{future sv},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var} N_{\text{SR tax},t+1}}. \quad (38)$$

When there is an unanticipated tax cut, either bond holders must suffer (i.e., returns, over the long run, are worse than expected prior to the tax cut), or future taxes must increase, or future spending must decrease. Which is it?

Table 6 reports results for a range of horizons, T, using the VAR that includes the tax-GDP ratio, as reported in Table 3. For comparability with previous tables, we collect the second and third terms on the right-hand side of (38), which capture adjustments to taxes and to spending, into a single column labelled "fiscal adjustment", and report the share of fiscal adjustment accounted for by the spending component in the column labelled "spending ratio." Again, the first three terms in each row would add up to precisely 100% if our loglinear approximation were exact. Bootstrapped 95% confidence intervals, calculated in the same way as in Table 2, are shown in square brackets.

In the very short run, at horizon $T = 1$, unexpected declines in tax are associated with unexpected contemporaneous *increases* in spending. This movement is in the "wrong" direction (hence the negative entry under fiscal adjustment in the first line) which exacerbates the shock to the fiscal position. At all horizons, returns contribute very little to resolving unexpected tax declines. As a result, over the long run, fiscal adjustment must pick up the slack: that is, an unexpected decline in tax today forecasts an increase in tax and a decrease in spending. The point estimates reported in the fourth column of Table 6 show that in the long run most of the adjustment takes place through increases in tax, with a smaller contribution from decreases in spending. That said, the confidence intervals are wide so our results are not decisive about the relative importance of tax and spending adjustment.

We can carry out a similar exercise for spending rather than taxes, rewriting the identity (37) as

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,x_{t+1}}_{\substack{\text{short-run spending news:}\\ N_{\text{SR spending},t+1}}} = -\frac{1-\beta}{\beta}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}}_{\text{return news: } N_{\text{return},t+1}} + \frac{1}{\beta}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j \Delta\tau_{t+1+j}}_{\text{tax news: } N_{\text{tax},t+1}} + \\ - \underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j \Delta x_{t+1+j}}_{\substack{\text{long-run spending news:}\\ N_{\text{LR spending},t+1}}} - \frac{1-\beta}{\beta(1-\rho)}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^T sv_{t+T}}_{\substack{\text{future fiscal position news:}\\ N_{\text{future sv},t+1}}}. \tag{39}$$

We write $N_{\text{tax},t+1}$ for the tax news term that appears on the right-hand side of identity (39). This is the sum of short-run tax news and long-run tax news, as defined in (37): $N_{\text{tax},t+1} = N_{\text{SR tax},t+1} + N_{\text{LR tax},t+1}$. Similarly, we write $N_{\text{SR spending},t+1}$ for short-run spending news and $N_{\text{LR spending},t+1}$ for long-run spending news, so that $N_{\text{spending},t+1}$ as defined after identity (37) is equal to the sum $N_{\text{SR spending},t+1} + N_{\text{LR spending},t+1}$.

We can now decompose the variance of short-run spending news as the sum of its covariances with news about returns, about tax growth, about long-run spending growth, and about the long-run fiscal position:

$$1 = \frac{\text{cov}\left(-\frac{1-\beta}{\beta}N_{\text{return},t+1}, N_{\text{SR spending},t+1}\right)}{\text{var}\ N_{\text{SR spending},t+1}} + \frac{\text{cov}\left(\frac{1}{\beta}N_{\text{tax},t+1}, N_{\text{SR spending},t+1}\right)}{\text{var}\ N_{\text{SR spending},t+1}} \\ + \frac{\text{cov}\left(-N_{\text{LR spending},t+1}, N_{\text{SR spending},t+1}\right)}{\text{var}\ N_{\text{SR spending},t+1}} + \frac{\text{cov}\left(-\frac{1-\beta}{\beta(1-\rho)}N_{\text{future sv},t+1}, N_{\text{SR spending},t+1}\right)}{\text{var}\ N_{\text{SR spending},t+1}}. \tag{40}$$

Table 7 reports results. We use the same format as before, collecting the second

Table 7: A variance decomposition for short-run spending news in postwar US data.

horizon	return	fiscal adjustment	future sv	spending ratio
1	-0.1	-15.1	116.5	0.0
	$[-0.1, -0.0]$	$[-18.8, -11.2]$	$[112.7, 120.2]$	$[0.0, 0.0]$
3	0.0	28.1	73.3	116.4
	$[-0.1, 0.2]$	$[-12.2, 60.0]$	$[41.6, 113.6]$	$[-246.9, 458.0]$
10	0.1	100.4	0.9	129.9
	$[-0.2, 0.4]$	$[59.0, 117.1]$	$[-15.8, 42.1]$	$[114.0, 188.4]$
∞	0.1	101.3	0.0	127.9
	$[-0.2, 0.5]$	$[100.9, 101.6]$	$[0.0, 0.0]$	$[113.2, 181.9]$

and third terms on the right-hand side of (40) into the single column labelled "fiscal adjustment" and reporting the share of spending in fiscal adjustment in the column labelled "spending ratio".

In the very short run, at horizon $T = 1$, unexpected increases in spending are associated with unexpected contemporaneous *decreases* in tax. Again, this movement is in the "wrong" direction, which exacerbates the shock to the fiscal position.

At longer horizons, we find once again that debt returns resolve almost none of the unexpected rise in short-run spending. As a result, fiscal adjustment must accomplish this; and fiscal adjustment is entirely driven by spending at long horizons. A positive spending news shock in the short run forecasts a large *decline* in long-run spending growth that more than offsets the original increase, as indicated by the entry greater than 100% in the column labelled "spending ratio."

5 International debt and deficits

We now repeat the analysis using a larger cross-section of countries for which we have been able to obtain appropriate data. We start by studying the UK, Canada, Switzerland and Japan, then turn to a collection of European countries that share the euro as currency.

The key challenge in international data is obtaining a time series for the market value of the government debt. Standard sources often report the face value of the debt instead. We have market value data from 1947 in the UK, 1989 in Canada, 1997 in Japan, and 1999 in Switzerland, as well as data for 11 countries in the eurozone (Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, and Portugal) since the creation of the euro in 1999. See appendix IA.1 for details. A secondary challenge is confirming the plausibility of imputed debt returns; we conduct this exercise in the appendix IA.3.

For the UK, Canada, Japan, and Switzerland, we follow the same procedure as in the US to estimate the linearization parameters ρ and β. When the mean surplus-debt ratio is positive, as it is in the UK, Canada, and Switzerland, we use it to infer ρ; when the mean surplus-debt ratio is negative, as it is in the US and Japan, we set $\rho = 0.999$. For the UK, $\rho = 0.976$; for Canada, $\rho = 0.960$; for Switzerland, $\rho = 0.970$.

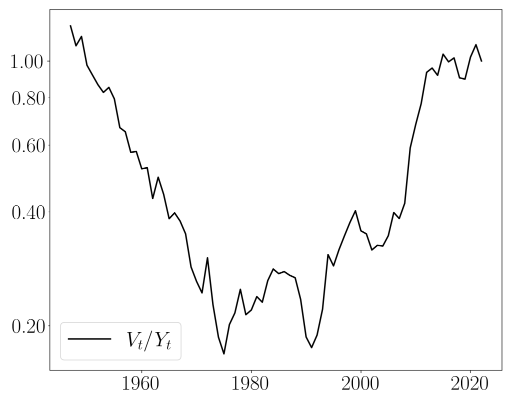
We choose β, conditional on ρ, to achieve the best least-squares fit of our fiscal position measure sv_t to $\log(1+S_t/V_t)$ which it approximates. For the eurozone countries, we impose a common value of $\rho = 0.997$, estimated from the panel of countries to reduce the number of free parameters that must be separately estimated in a short sample and to reflect fiscal constraints that apply in a similar manner to all eurozone governments. We then choose β separately for each country to achieve the best least-squares fit in each country. The implied linearization parameters are reported in appendix section IA.5.

5.1 The UK, Canada, Japan, and Switzerland

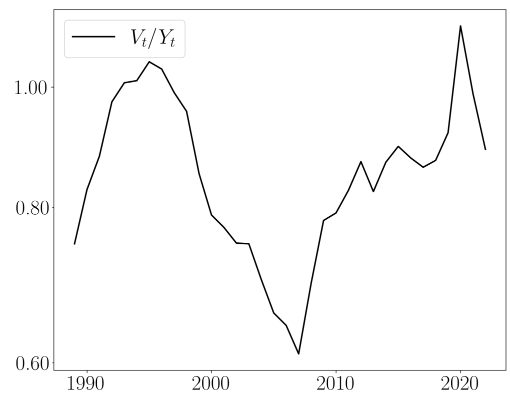
Figure 6 plots the history of debt-GDP ratios in the UK, Canada, Japan, and Switzerland. The nonstationarity of these ratios is visually apparent, and confirmed by unit root tests in internet appendix section IA.4. Figure 7 plots sv_t and $\log(1 + S_t/V_t)$ for the same four countries. The figure illustrates the stationarity of the fiscal position sv_t and the accuracy of its loglinear approximation of $\log(1 + S_t/V_t)$.

Section IA.7.1 of the internet appendix reports VAR estimates for each of the four countries. Table 8 reports the implications of these estimates for the variance decomposition of the fiscal position at a 10-year horizon for each country. Most of the patterns we saw in US data appear in these other four countries as well. Returns on government debt have a minimal influence on the dynamics of the fiscal position, and the fiscal position mean-reverts quickly enough that in all countries at least three-quarters of its variability is accounted for by ten-year fiscal adjustment. In the UK, Canada, and Switzerland, fiscal adjustment takes place primarily through adjustment of the growth

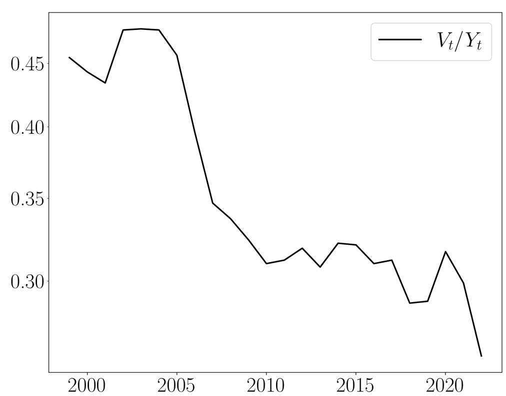
Figure 6: Debt-GDP ratios in the UK, Canada, Japan, and Switzerland



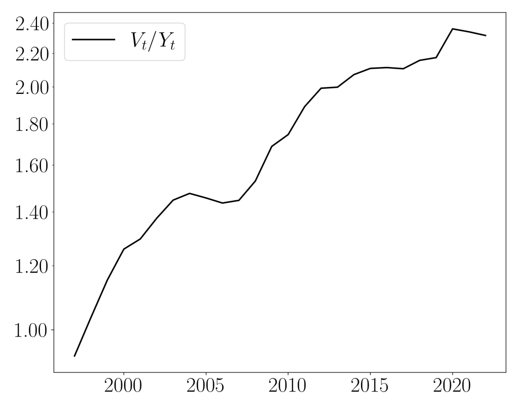
(a) UK



(b) Canada

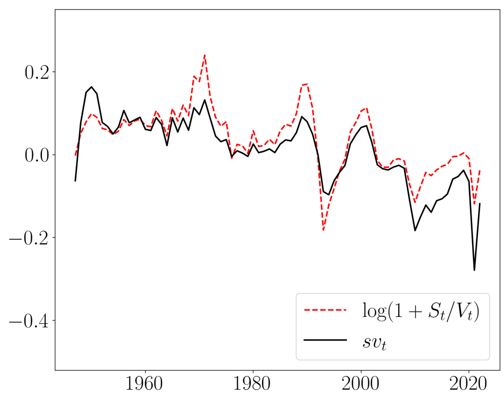


(c) Japan

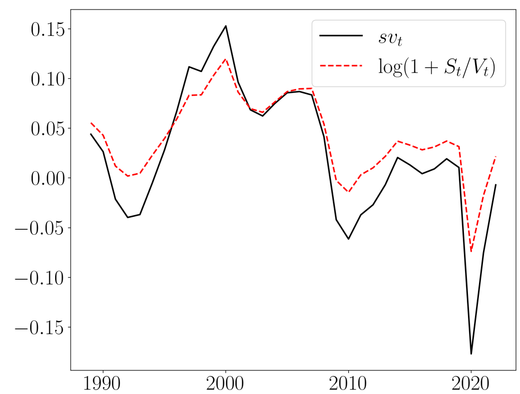


(d) Switzerland

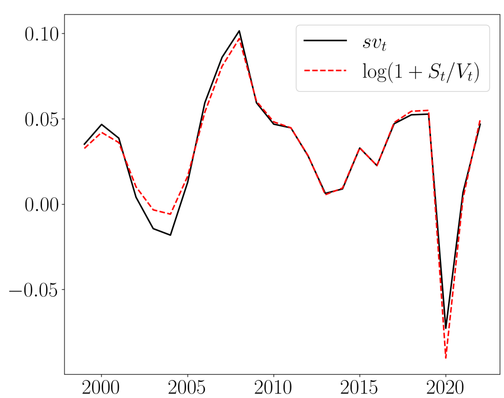
Figure 7: Surplus-to-debt ratios in the UK, Canada, Japan, and Switzerland



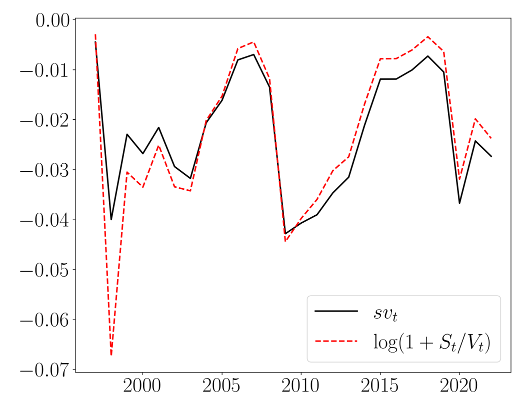
(a) UK



(b) Canada



(c) Japan



(d) Switzerland

Table 8: Variance decomposition of the fiscal position sv_t for the UK, Canada, Japan, and Switzerland at horizon $T = 10$, based on the VAR system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$

country	return	fiscal adjustment	future sv	spending ratio
UK	1.5	86.4	13.4	105.2
	$[-3.1, 6.6]$	$[49.3, 104.8]$	$[-1.9, 48.1]$	$[37.6, 213.1]$
Canada	3.7	97.8	1.6	78.1
	$[0.2, 8.4]$	$[50.9, 104.0]$	$[-4.3, 47.0]$	$[51.1, 163.8]$
Japan	-0.2	77.6	26.8	-25.6
	$[-0.8, 0.2]$	$[1.9, 134.9]$	$[-31.0, 102.5]$	$[-461.3, 75.1]$
Switzerland	2.6	104.6	-2.7	78.5
	$[-0.1, 6.4]$	$[84.9, 131.5]$	$[-29.7, 16.5]$	$[34.4, 140.1]$

rate of government spending. This is not the case in Japan, however, where more than all the adjustment is accounted for by variation in the growth rate of tax revenue.

Table IA.22 in the Appendix reports results for the UK based on local projections at 1, 3, 10-year horizons. (Given the 10-year horizon, we require a long sample period for the local projections approach to be feasible. The UK is the only one of the four countries for which we observe data over a sufficiently long period.) The UK fiscal position mean-reverts substantially over 10 years, so that the future fiscal position contributes less than half the variance of the fiscal position at this horizon. Consistent with our other results, returns contribute very little to the variance of the fiscal position, and fiscal adjustment is dominated by spending (although the standard error for the spending ratio is extremely wide).

5.2 The euro area

Finally we consider 11 Eurozone countries in the years since the creation of the Euro in 1999. Section IA.7.2 of the internet appendix reports VAR estimates for each of these 11 countries. Table 9 reports the implications of these estimates for the variance decomposition of the fiscal position at a 10-year horizon for each country. Once again we see similar patterns to those we have described in US data. Returns on government debt have a minimal influence on the dynamics of the fiscal position, and the fiscal position mean-reverts quickly enough that almost all its variability is accounted for by ten-year

Table 9: Variance decomposition of the fiscal position sv_t for 11 eurozone countries at horizon $T = 10$, based on the VAR system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$

country	return	fiscal adjustment	future sv	spending ratio
Austria	1.3	106.1	−2.8	26.5
	[0.4, 6.1]	[62.2, 152.4]	[−49.2, 39.2]	[−156.5, 57.9]
Belgium	1.2	102.7	0.7	64.3
	[0.3, 3.3]	[47.7, 116.6]	[−14.3, 55.1]	[11.7, 179.9]
Germany	0.3	104.9	−0.7	77.0
	[−0.3, 1.4]	[77.9, 133.4]	[−29.1, 26.2]	[53.8, 175.5]
Spain	−0.1	106.1	−1.5	81.9
	[−1.4, 1.1]	[36.2, 149.3]	[−43.9, 68.7]	[18.9, 319.8]
Finland	1.1	97.1	6.3	85.8
	[0.6, 2.0]	[37.3, 112.4]	[−8.7, 65.5]	[47.7, 321.7]
France	1.1	103.2	0.3	26.7
	[−0.0, 3.5]	[32.4, 131.8]	[−29.3, 71.3]	[−28.2, 97.2]
Greece	0.5	108.8	−4.7	121.0
	[−1.0, 2.7]	[47.0, 145.9]	[−42.1, 56.1]	[42.9, 289.8]
Ireland	0.1	104.7	−0.3	117.0
	[−0.5, 0.8]	[56.7, 121.2]	[−16.9, 47.6]	[70.3, 360.6]
Italy	0.7	106.6	−2.7	60.5
	[−0.2, 2.4]	[68.9, 144.6]	[−41.1, 34.3]	[−10.8, 125.3]
Netherlands	0.2	104.4	−0.0	82.0
	[−0.5, 0.7]	[81.6, 119.8]	[−15.5, 22.8]	[48.6, 200.5]
Portugal	−0.5	103.4	1.6	82.4
	[−2.2, 0.3]	[50.7, 110.3]	[−5.5, 54.7]	[31.6, 200.8]

fiscal adjustment. In all countries except Austria and France, fiscal adjustment takes place primarily through adjustment of the growth rate of government spending. We note however that confidence intervals for the relative contributions of spending and taxes are quite wide in the Eurozone data.

6 Conclusion

Conventional tests do not reject the presence of a unit root in the debt-GDP ratio in postwar US data. We have presented a framework for fiscal analysis that takes this uncomfortable fact into account by making the surplus-debt ratio—which does appear to be stationary—the central object of interest.

Our framework considers not only what one might call the burden of the debt—that is, the size of the surplus that is required to service the debt—but also the size of the government relative to the debt. Both tax revenue and government spending are typically very large relative to the primary surplus which is the difference between these two numbers. Thus, say, a 1% change in the level of spending can have a very large proportional impact on the primary surplus. This has important implications for fiscal adjustment.

We analyze the contributions of taxes and spending to surplus separately, and so we can distinguish between, say, declines in tax revenue and increases in government expenditure. There are good economic reasons to analyze these two variables separately: in a recession, tax revenue declines at a faster rate than GDP in the presence of increasing marginal tax rates, whereas spending increases, but there is no particular reason to expect tax and spending to adjust symmetrically. Concretely, we find that despite the nonstationarity of the surplus-GDP ratio and the expenditure-GDP ratio, the US tax-GDP ratio does appear to be stationary, a fact that has important implications for our analysis of US data.

We organize our empirical work by deriving a loglinear approximation to the surplus-debt ratio that summarizes the fiscal position of the government. Our key identity relates the fiscal position to future returns on government debt and to future tax and spending growth rates, just as the identities derived by Campbell and Shiller (1988) relate the dividend yield on a security to that security's future returns and dividend growth rates. A weak fiscal position must be followed by some combination of low long-run returns on government debt, high long-run tax growth, and low long-run spending growth.

We use this identity to interpret variation in the fiscal position over time in postwar data from the US and from 15 other developed countries. In all these countries the fiscal position has limited forecasting power for future debt returns over the long run; instead, it forecasts long-run future fiscal adjustment, i.e., changes in the growth rates of tax revenue and government spending. In the US and in most other countries we study, with the notable exception of Japan, fiscal adjustment occurs more through spending growth than through growth of tax revenue.

These findings contrast with the results of papers that study the ratio of debt to GDP, a nonstationary ratio that has little ability to predict fiscal adjustment and mostly predicts its own future value (Jiang et al. (2021*b*)). These findings also differ sharply from those reported in the literature that carries out variance decompositions for stock market returns, following John Y. Campbell (1991), where it is generally argued that valuation ratios have more forecasting power for returns than for cashflow growth.

We also use our identity to analyze long-run responses to tax and spending shocks. Again we find that debt returns, both unexpected returns at the time the shocks occur and subsequent predictable returns, play almost no role in these responses. Instead, mean-reverting tax and spending growth satisfy the government's intertemporal budget constraint allowing debt value to remain stable. While our framework does not allow us to say which variables are exogenous and which are endogenous, this pattern does tell us that if, as the fiscal theory of the price level asserts, debt value is endogenous, postwar governments in the US and 15 other developed countries have chosen fiscal policies that avoid large predictable or unpredictable returns to debtholders.

One reason for these policy choices could be that large swings in the value of the debt are politically risky for incumbent policymakers. As James Carville, a political adviser to Bill Clinton, is reported to have said, "I used to think that if there was reincarnation, I wanted to come back as the president or the pope or as a .400 baseball hitter. But now I would like to come back as the bond market. You can intimidate everybody." An illustration of this principle was recently provided by the market reaction to unexpectedly large tax cuts in the September 2022 "mini-budget" in the United Kingdom, which led to the rapid departure of both the Chancellor of the Exchequer and Prime Minister Liz Truss.

It is possible, perhaps even probable, that our framework would attribute a more significant role to debt returns in countries that have experienced turbulent macroeconomic crises. A priority for future research should be to apply our analysis to other countries, including emerging markets, where data are available on the market value (as opposed to the face value) of the public debt.

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