Green Multistakeholderism*

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Abstract

We analyze how firms and groups of stakeholders - such as creditors and workers - form teams in multi-sided markets, given their heterogeneous productivity and non-pecuniary preferences for abatement. Their sorting takes into account production complementarities and that emission increases with production. In contrast to models with homogeneously-productive stakeholders, firms in general are not indifferent between being green or brown. Therefore, exit and engagement can both arise in equilibrium, depending on competition and prices. Greeniums—the earnings of brown versus green stakeholders—reflect not just the standard compensating differentials but also sorting effects. We show that multistakeholder heterogeneity is needed to rationalize a number of salient features of the data.

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1 Introduction

A range of stakeholders increasingly demand that firms abate their emissions as a precondition for working together. Examples include investors pledging to hold green stocks (The Net-Zero Asset Managers Initiative), banks committing to green loans (United Nations Net-Zero Banking Alliance), workers threatening their firms with strikes unless they carbon neutral (Amazon Employees for Climate Justice), and corporate partners like Apple requiring suppliers be sustainable (Wall Street Journal (2022)). This has led to a shift away from a shareholder to a multistakeholder capitalism (Business Roundtable (2019)), in which firms face a constrained profit-maximization problem that needs to take into account sustainability goals or abatement preferences of all stakeholders.

We develop a theory of how green stakeholders incentivize their firms to abate their emissions. There are multiple groups of stakeholders and each group is required for output. Heterogeneously productive firms and skilled stakeholders from each group optimally sort into teams depending on output complementarities, the sharing of abatement costs and the non-pecuniary benefits of abatement to green stakeholders. Starting with the same emissions intensity from output, costly firm abatement is an endogenous outcome that increases with the number of green stakeholders on its team.

Since firms and multiple groups of stakeholders have to form teams in order for there to be production and each group of stakeholder can differ in their skills and green preferences, the matching problem here can be understood as multilateral agents or multi-sided markets with multidimensional characteristics and transferable utilities. All agents, firms and stakeholders, take the equilibrium utilities of the others as given and transfers are used to clear markets.

The literature on multilateral matching markets mainly focuses on the existence and efficiency of the equilibrium (e.g., Hatfield and Kominers 2015), while we are interested in the full characterization under multiple attributes. Unlike one-dimensional matching models (Sattinger 1979, Tervio 2008, Gabaix and Landier 2008), where the supermodularity of the surplus function simply governs the sorting condition — it is well known that the conditions are generally more complex in the multidimensional content. Previous work with multidimensional matching (Dupuy and Galichon 2014, Lindenlaub 2017, Chiappori, McCann, and Pass 2016, Chiappori, Oreffice, and Quintana-Domeque 2018) have provided characterization only with two-sided markets and using specific properties and assumptions that do not apply to our setting.

We are able to characterize the unique stable matching equilibrium by developing a novel iterative solution procedure. Groups of stakeholders are ranked by their measure of green preferences in descending order. The matching equilibrium with just two highest ranked groups, say workers and investors, is obtained. The matching equilibrium between this team that has just been formed and the next type of stakeholder, say banks, is then obtained. We keep iterating until we have matched all types of stakeholders before then obtaining the matching equilibrium between these teams and firms.

The optimal allocation of teams is determined by three forces. The first is that conditional green preferences, the most productive firms match with the skilled stakeholders from each group. The second is that green stakeholders derive disutility from the emissions of the firms that they are matched with. Once a firm pays for abatement, there are economies of scale from having more green stakeholders, i.e. the sharing of abatement costs. The third is that green stakeholders would have to sacrifice more wages to turn a more productive firm green. As a result, differences in the earnings of stakeholders in green firms versus brown firms reflect not only compensating differentials, that is abatement costs, but also sorting effects.

Balancing these three forces, a prominent feature of the optimal-teams equilibrium is that green stakeholders of comparable productivity want to sort into the same teams and with less productive firms than they otherwise would absent green preferences. There are also rents to brown stakeholders as green stakeholders are matched with less productive firms. Since there will in general be an imbalance of green preferences across different stakeholder groups, it is optimal for firms above a certain productivity cut-off to be purely green or brown. There must then be mixing for the remaining stakeholders.

Our model matches salient facts that are difficult to rationalize in the literature. First, green preferences lead to abatement through both exit and engagement channels in our model. Broccardo, Hart, Zingales, et al. 2022 highlight the downside of exit compared to engagement through voting. But our model suggests that the effectiveness of engagement depends on stakeholder productivity. More productive stakeholders that are previously matched to large firms need to exit to rematch with less productive firms. In contrast, less productive stakeholders can effectively engage with their existing firm and turn it green.

Second, stakeholder impact differs by firm size. The ability of green stakeholders to turn large firms green is more limited than turning small firms green. Third, stakeholder impact differs across groups, e.g. banks versus workers. Suppose there are fewer green banks than green workers. Increasing the measure of green banks would not lead to more green firms on the extensive margin since the green banks would just sort with existing green teams of workers.

Fourth, and as a result, greeniums, the difference in earnings of brown versus green stakeholders, vary across groups. Fifth, exit leads to the remaining brown firms polluting more, an effect that is absent in existing models of exit by institutional investors (see, e.g., Heinkel, Kraus, and Zechner 2001, Hong and Kacperczyk 2009, Pástor, Stambaugh, and Taylor 2021, Pedersen, Fitzgibbons, and Pomorski 2021).

In our quantitative analysis, given that data on the preferences of stakeholders is missing, we develop a test of this sorting using emissions and revenues data. Using our model, we can map a firm's emissions holding fixed its revenue to the number of green stakeholders on a firm's team. Under a counterfactual benchmark in which stakeholders and firms are matched randomly, we show that the distribution of the count of green stakeholders on a team should be Poisson. Under our sorting model, the count distribution should be over-dispersed, which we verify is indeed the case using emissions and revenue data from the utilities sector.

2 Model

Production and carbon emissions. Production requires a joint participation from N types of stakeholders and firm. Stakeholders of type $\ell \in \{1, 2, ..N\}$ have different skills which affect output and are summarized by x_ℓ. For example, one can interpret type 1 (type 2) as banks (workers) and x_1 as bank loan size (x_2 as the talent of the worker). We use $\ell = N+1$ to denote firms, which have productivity x_{N+1}. There are thus $N+1$ types of agents, $\ell \in L \equiv \{1, 2..., N+1\}$ in this economy, including the firm. Each type of stakeholder or firm has a unit mass, with smoothly distributed skills. That is, the distribution of x_ℓ has continuous finite support without gaps, which is denoted by $X_\ell \; \forall \ell \in L$.

The total production of a firm depends on the skills of its stakeholders and its own productivity, summarized by the vector $\boldsymbol{x} = (x_1, ..x_N, x_{N+1})$. For tractability, we further assume that the production function is multiplicatively separable, which is given by

$$y(\boldsymbol{x}) = \prod_{\ell=1}^{N+1} x_\ell.$$

As discussed in Tervio (2008), the linear assumption on the arguments does not preclude different stakeholders having different skills contributing to their ability to affect output.[1]

Given any production y, the emission is given by σy. Thus, the emission rate (without abatement) is σ for all firms. All firms have access to an abatement technology which can reduce their own emissions at a linear cost c.

Green stakeholders. Stakeholders can only join one firm. This indivisibility assumption captures the notion of a relationship. For example, it means that a bank with capital size x_ℓ will choose one particular firm to lend to rather than distributing loans across all firms. In this sense, each stakeholder has a relationship in a specific firm, which is why we refer them as the stakeholders of that firm in the first place.

We model green stakeholders as the ones that prefer their own firms to have lower carbon emissions. Let $\theta_\ell \in \{0, 1\}$ indicate the green ($\theta_\ell = 1$) vs. brown ($\theta_\ell = 0$) stakeholders of type ℓ. The utility of a stakeholder depends on the monetary compensation that he receives and, if

[1]Specifically, one could interpret $x_\ell = b_\ell(\hat{x}_\ell)$, which represents the effective ability of some underlying skill $\hat{x}_\ell$, where b_ℓ is an increasing transformation of the scale of measurement for a factor quality. For example, a Cobb-Douglas production function $x_0 \hat{x}_1^\alpha \hat{x}_2^{(1-\alpha)}$ can be nested as $x_1 = \hat{x}_1^\alpha$ and $x_2 = \hat{x}_2^{1-\alpha}$.

he is a green stakeholder, he has disutility over the emission produced by his firm, which yields

$$u(p, e|\theta) = p - \theta\psi(e).$$

Here p denotes the monetary compensation, and e denotes the emission produced by his firm. In other words, green stakeholders are the ones with non-pecuniary preference over emissions. We assume that $\psi'(e) > 0$ and $\psi''(e) > 0$. We normalize the outside options of type ℓ stakeholders to be zero.

Characteristics for stakeholders are thus generally two-dimensional, denoted by $a_\ell \equiv (x_\ell, \theta_\ell) \subseteq A_\ell \equiv X_\ell \times \{0, 1\}\ \forall \ell$. They are distributed according to probability measures μ_ℓ on A_ℓ, and where the total measure of green stakeholders is denoted by λ_ℓ. Firms, on the other hand, are risk-neutral and profit-maximizing and do not have green preference themselves. Firms can be interpreted as an important type of stakeholder — manager or founder. That is, $a_{N+1} = (x_{N+1}, 0)$ $\forall x_{N+1}$. In other words, firms are special in the sense that their characteristics are always one-dimensional ($\lambda_{N+1} = 0$).

Team formation. We look for a competitive equilibrium, where each stakeholder chooses his team optimally, taking the equilibrium utility for other stakeholders as given. That is, if a stakeholder wants to attract another stakeholder (x_ℓ, θ_ℓ), he must provide the equilibrium utility, denoted by $U_\ell(x_\ell, \theta_\ell)$, for that stakeholder. In the case when he wants to match with a brown stakeholder, who only cares about the transfer, then this is the same as saying that the stakeholder takes the prices (such as wages and interest repayments) as given when attracting a brown worker or investor. On the other hand, when a firm wants to hire a green stakeholder, whose utility also depends on the emissions, then the firm must provide the bundle of the fee and abatement such that the green stakeholder is willing to come.

We first start with firm's problem, who choose their stakeholders $a_\ell = (x_\ell, \theta_\ell)$ for all types ℓ as well as abatement decision, denoted by m, to maximize their profits, taking the equilibrium utility for stakeholders as given. The equilibrium utility for a brown firm can thus be expressed as

$$U_{N+1}(x_{N+1}, 0) = \max_{\left\{(a_\ell, p_\ell)_{\forall \ell}, m \geq 0\right\}} y(\boldsymbol{x}) - cm - \Sigma_{\ell=1}^{N} p_\ell, \tag{1}$$
$$s.t. p_\ell - \theta_\ell \psi\left(\sigma y(\boldsymbol{x}) - m\right) \geq U_\ell(x_\ell, \theta_\ell) \, \forall \ell \leq N,$$

where p_ℓ represents the transfer to stakeholder ℓ. The constraint says that if a firm wants to hire a stakeholder of type (x_ℓ, θ_ℓ), the utility that he provides to the stakeholder, which generally depends on the fees p_ℓ and firm's emission $\sigma y(\boldsymbol{x}) - m$, must be larger than the equilibrium utility of that stakeholder.

Note that, due to the transferable utilities, the firm must choose the optimal abatement to maximize the joint payoff given any matches. This also means who makes the abatement decision does not matter for the result. Let $\Lambda(\boldsymbol{a}) \equiv \Sigma_{\forall \ell \in L} U(a_\ell)$ denote the total matching surplus generated by the team $\boldsymbol{a} = (a_1, a_2, .., a_N, a_{N+1})$, which includes the set of stakeholders of all types together with the firm $a_{N+1} = (x_{N+1}, 0)$. The abatement decision within any match must solve

$$\Lambda(\boldsymbol{a}) = \max_{m \geq 0} y(\boldsymbol{x}) - cm - n(\boldsymbol{\theta})\psi(\sigma y(\boldsymbol{x}) - m), \tag{2}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2.., \theta_N, \theta_{N+1})$ is the vector that summarizes stakeholders' green preference and $n(\boldsymbol{\theta}) \equiv (\Sigma_{\forall \ell \in L} \theta_\ell)$ represents the number of green stakeholders within the match, which we refer to as green index of the team. Clearly, a firm will not have an incentive to pay for the abatement if it only matches with brown stakeholders $n(\boldsymbol{\theta}) = 0$.

More generally, for any stakeholder a_ℓ, when choosing the optimal team, he will choose the one with characteristics $a_{\ell'}$ from all types except his own $\forall \ell' \in L \setminus \{\ell\}$. His optimal team decision can thus be expressed as

$$U_\ell(a_\ell) = \max_{\{a_{\ell'}\}_{\forall \ell' \in L \setminus \{\ell\}}} \Lambda(\{a_{\ell'}\}_{\forall \ell' \in L \setminus \{\ell\}}, a_\ell) - \Sigma_{\ell' \in L \setminus \{\ell\}} U(a_{\ell'}). \tag{3}$$

That is, the payoff to a stakeholder a_ℓ is the total surplus minus the equilibrium utilities to other stakeholders in the team. One can also see that the firm decision is just a special case of Equation 3 where $\ell = N + 1$. While we assume that the firm is paying the abatement costs in Equation 1, all stakeholders are effectively sharing such costs through transfers.

Competitive equilibrium. Let γ be a probability measure on the $N + 1-$fold Cartesian product which represents the matching between firm x_{N+1} and the set of stakeholders $\boldsymbol{a} = (a_1, a_2, ..a_N)$, and where the marginal distribution of a_ℓ is μ_ℓ $\forall \ell$. That is, the market clearing condition is satisfied.

Definition 1. An equilibrium consists of equilibrium utility $U_\ell(a_\ell)$ for stakeholder a_ℓ, the allocation γ and abatement $m^*(x_{N+1})$ such that if $\gamma(\boldsymbol{a}, x_{N+1}) \in$ spt γ, then $\{\boldsymbol{a}, m^*(x_{N+1})\}$ solves Equation (2) and $U_\ell(a_\ell)$ is given by Equation (3). And the matching γ satisfies the market clearing condition.

Theoretically, our setting can be understood as multilateral matching with transfers, and it is known that a competitive equilibrium exists, and corresponds to stable outcomes (Hatfield and Kominers (2015)).

Remarks on green preferences. We assume that green stakeholders only care about the emission level produced by their own firm. This assumption has two implications. First, our economic environment does not have externality in the sense that the abatement of a firm does not directly affect others outside of that firm. Nevertheless, a stakeholder's disutility of emissions $\psi(e)$, which is exogenous, can potentially depend on the aggregate stock of emissions. Hence, if a green stakeholder's demand of the abatement increases with aggregate emissions, the aggregate stock of emissions can affect the equilibrium outcomes.

Second, since a stakeholder's disutility is on the level of emissions, a firm that produces more effectively faces a higher cost when hiring a green stakeholder, as it has to abate more. Thus, in our setting, abatement decision differs from the traditional amenities because more productive firms produce more emissions (or social ills). For this reason, there is an inherent trade-off between abatement and productivity, which does not typically present in traditional amenities settings. Because of this trade-off, the 2-dimensional sorting problem on productivity and green preferences are naturally intertwined.[2]

Optimal abatement given any match. Given any matches between x_{N+1} and the set of stakeholders $\boldsymbol{a}$**,** the firm will choose the abatement decision optimally to maximize the joint payoff. Let $\Lambda(\boldsymbol{a},x_{N+1})$ denote the joint payoff $(U_{N+1}(x_{N+1},0)+\sum_{l=0}^{N}U_l(a_l))$ when the firm x_{N+1} is hired with the set of stakeholder $\boldsymbol{a}$. Given the matches, the optimal abatement decision thus solves

$$\Lambda(\boldsymbol{a},x_{N+1})=\max_{m\geq 0}y(\boldsymbol{x})-cm-n(\boldsymbol{\theta})\psi(\sigma y(\boldsymbol{x})-m)-\Sigma_{l=1}^{N}u_l^0,$$

where $\boldsymbol{\theta}=(\theta_1,\theta_2..,\theta_N)$ is the vector that summarizes stakeholder's green preference and $n(\boldsymbol{\theta})\equiv\left(\Sigma_{\ell=1}^{N}\theta_\ell\right)$ represents the number of green stakeholders within the match.

3 Properties of Optimal Teams

We now proceed to analyze the surplus function and matching outcomes.

Joint surplus given any match. Observe from Equation 2, the surplus function of a team can be conveniently summarized by two variables: productivity $y(\boldsymbol{x})$ and green index $n(\boldsymbol{\theta})$. Thus, $\Lambda(\boldsymbol{a})=\Omega(y(\boldsymbol{x}),n(\boldsymbol{\theta}))$, where

$$\Omega(y,n)=\max_{e\leq\sigma y}\left\{y-c(\sigma y-e)-n\psi(e)\right\}. \tag{4}$$

Given the emissions σy, a higher abatement simply means lower emissions as $e=\sigma y-m$. Firms will have no incentives to abate without any green stakeholders ($n=0$). We focus on

[2]To be concrete, consider an alternative formulation that shuts down the dependence between productivity and abatement, where

$$\Lambda(\boldsymbol{a})=\max_{m\geq 0}y(\boldsymbol{x})-cm-n(\boldsymbol{\theta})\psi(\sigma-m).$$

A natural interpretation of this setting is where a firm can provide costly amenities (such as office amenities) that improves workers' utilities. In this case, one can see the surplus is separable in production y and the green index n; hence, the sorting of between ability (which only affects production) and green preference (which only affects green index) can be solved independently.

the case where the optimal solution of the abatement is interior for all firms that have green stakeholders. Given the number of green stakeholders n, a firm will reduce its emission to the optimal level, where $\xi_n^* \equiv \arg\max_{e \leq \sigma y} \{y - c(\sigma y - e) - n\psi(e)\}$ which solves

$$n\psi'(\xi) = c \tag{5}$$

whenever the interior solution exists. That is, the marginal improvement in green stakeholder's utilities by reducing emission equals the marginal cost of abatement.

Clearly, the optimal emission ξ_n^* decreases in n. Moreover, due to the linear abatement cost, the optimal emission (after abatement) is independent of productivity y. In other words, for any $n \geq 1$, the optimal abatement for a firm with productivity y and n green stakeholders is given by

$$m^*(y, n) = \sigma y - \xi_n^*, \forall n \geq 1, \tag{6}$$

which increases in y (i.e., a firm produces more emission will then also abate more) and the green index n. The surplus function can be further simplified to the following expression:

$$\Omega(y, n) = \left\{ \begin{array}{cc} y & n = 0 \\ (1 - c\sigma)y + c\xi_n^* - n\psi(\xi_n^*) & n \geq 1 \end{array} \right\}. \tag{7}$$

We proceed to characterize the equilibrium under the following parameter assumptions, which guarantee that (1) abatement is given by the interior solution for any $n \geq 1$ and (2) the production generates positive surplus in spite of abatement. Let $\underline{x}_\ell$ represent the lowest skill of type ℓ agent and the least productive combination is thus given by $\underline{\boldsymbol{x}} \equiv (\underline{x}_1, ..\underline{x}_N, \underline{x}_{N+1})$.

Assumption 1. *(1) (interior abatement)* $\psi'(\sigma y(\underline{\boldsymbol{x}})) > c$ *and (2) (positive surplus)* $(1 - c\sigma) > 0$ *and*

$$(1 - c\sigma)y(\underline{\boldsymbol{x}}) + (c\xi_N^* - N\psi(\xi_N^*)) > 0.$$

The general matching problem here involves multi-agents and multi-dimensional characteristics. To proceed, we first establish two important concepts for the matching outcomes, and then we apply these principles to construct the optimal matches.

3.1 Sorting on Green Preferences

We first analyze the effect of green preferences on sorting outcomes while fixing the productivity of the match. When two stakeholders only differ in their green preferences and their teams have the same productivity index, then green agents must be in a coalition that has a higher green index.

Lemma 1. *(Concentration of Green Stakeholders) Consider two type stakeholders of type ℓ with the same ability ($x_\ell = x_\ell$) but different green preference, then $n^*_{-\ell}(x_\ell, 1) \geq n^*_{-\ell}(x_\ell, 0)$.*

Observe that $\Omega(y, n)$ is decreasing and strictly convex in n. Thus, for any $n' > n$, we have

$$\Omega(y, n) + \Omega(y, n' + 1) > \Omega(y, n + 1) + \Omega(y, n'),$$

where LHS (RHS) represents the surplus when adding the green agent the group that has a higher (lower) green index. Conditional on the productivity of the matches among the rest of the stakeholders, it can never be the case that the green stakeholder is matched with the firm with a lower green index than the equivalent brown stakeholder. Intuitively, since green stakeholders derive disutility of the emissions of the firms that they are matched with, there are economies of scale for abatement.

3.2 Sorting on Productivity

Let $(y_{-\ell}, n_{-\ell})$ denote the productivity and green index of the team $\{a_{\ell'}\}_{\forall \ell' \in L \setminus \{\ell\}}$ that excludes agent of type a_ℓ. For the stakeholder (x_ℓ, θ_ℓ), the surplus when matching with the team with characteristics $(y_{-\ell}, n_{-\ell})$ can thus be expressed as $\Omega\left(y_{-\ell} x_\ell, (n_{-\ell} + \theta_\ell)\right)$.

Conditional on being a green stakeholder, due to the complementarity in the production function, a green stakeholder with higher x_ℓ must thus be matched with a team with higher productivity $y_{-\ell}$ as

$$\frac{\partial\Omega\left(y_{-\ell}x_\ell, n_{-\ell}+1\right)}{\partial x_\ell} = (1-c\sigma)y_{-\ell}. \tag{8}$$

That is, the marginal value of skill of a green stakeholder is $(1-c\sigma)y_{-\ell}$.

On the other hand, the marginal value of having a skilled brown stakeholder is given by,

$$\frac{\partial\Omega\left(y_{-\ell}x_\ell, n_{-\ell}\right)}{\partial x_\ell} = z_{-\ell}(y_{-\ell}, n_{-\ell}) \tag{9}$$

where the z index is defined as

$$z_{-\ell}(y_{-\ell}, n_{-\ell}) \equiv \begin{cases} (1-c\sigma)\, y_{-\ell} & n_{-\ell} \geq 1 \\ y_{-\ell} & n_{-\ell} = 0 \end{cases} \tag{10}$$

Productivity discounts due to abatement. Observe that whenever the firm abates, any marginal increase in productivity Δy requires additional abatement cost $c\sigma\Delta y$. Hence, the marginal value of stakeholder skills to surplus is discounted by the factor of $(1-c\sigma)$ relative to the case without abatement. Moreover, this discount $(1-c\sigma)$ is a constant and independent of the green index of the team $n_{-\ell}$. Why? Under the assumption of linear abatement costs c, the optimal abatement decision is separable in y and n, $\forall n \geq 1$, as shown in Equation 5.

Brown and green stakeholders can value productivity differently. The only way to avoid abatement is to have a pure brown team. Hence, this can only happen when a brown stakeholder joins a team without any green stakeholders. As a result, the marginal value of having a skilled brown stakeholder is particularly high for a brown team (i.e., when $n_{-\ell} = 0$.)

A brown team with lower productivity could have the same z index as a green team with a higher productivity $y'_{-\ell} > y_{-\ell}$, where $y_{-\ell} = (1-c\sigma)y'_{-\ell}$. A brown stakeholder is relatively valuable to a pure brown team ($n_{-\ell} = 0$), as matching with a brown stakeholder allows them to avoid costly abatement.

This also means that if a brown stakeholder is indifferent between teams with different $n_{-\ell}$, these teams must have the same $z_{-\ell}$. In particular, the team with green stakeholders must be more productive to compensate for the abatement costs. However, this ranking distortion does not exist for green stakeholders since his matching teams are subject to abatement costs anyway.

Let $y^*_{-\ell}(a_\ell)$, $n^*_{-\ell}(a_\ell)$, and $z^*_{-\ell}(a_\ell)$ denote the characteristics of the optimal matching team for agent a_ℓ. By monotone comparative statics, Equations 8 and 9imply that a better skilled green (brown) agent must be matched with a team with higher $y_{-\ell}$ ($z_{-\ell}$).

Lemma 2. *Conditional on green preference* ($\theta_\ell = 1$), *if* $x'_\ell \geq x_\ell$, *then* $y^*_{-\ell}(x'_\ell, 1) \geq y^*_{-\ell}(x_\ell, 1)$. *Conditional on brown preference* ($\theta_\ell = 0$), *if* $x'_\ell \geq x_\ell$, *then* $z^*_{-\ell}(x'_\ell, 0) \geq z^*_{-\ell}(x_\ell, 0)$.

Sorting on productivity only under shareholder capitalism. To understand how green preference affects matching, it is useful to compare to the standard setup where the matching simply maximizes the aggregate production. This can be interpreted as the outcome when all stakeholders and firms are brown (i.e., the shareholder capitalism benchmark), or, equivalently, green preference is realized only after matching. In this case, due to the complementarity in the production function, there will be simply positive sorting for all factors $x_\ell \; \forall \ell \leq N+1$. That is, a stakeholder of type ℓ with ability ranking i will thus be matched with a stakeholder of type ℓ' with the same ranking and the same green preference. The revenue for firms at the ranking i is given by $y[i] = \prod_{\ell=1}^{N+1} x_\ell[i]$, where $x_\ell[i]$ denote the ability of an i quantile for type ℓ agent.[3]

4 Symmetric Multistakeholder Impact

We now study how heterogeneous green preference and productivity together affect the sorting. To fully characterize the matching outcome, we assume that the distribution of skills and green preferences are independent and identically distributed throughout the rest of the paper for simplicity.

[3]That is, let $F_\ell(x_\ell)$ represents the measure of stakeholder of type ℓ with ability no larger than x_ℓ, $x_\ell[i]$ is defined by $x_\ell[i] = x_\ell \; s.t. F_\ell(x_\ell) = i$.

Assumption 2. *Skills and green preferences are independently distributed. For any level of skill x_ℓ, the probability of being a green stakeholder of type ℓ is λ_ℓ.*

Moreover, in this section, we start with a special case where the percentage of green stakeholders in a group is the same for all groups of stakeholders: $\lambda_\ell = \lambda \, \forall \ell \leq N$.

4.1 Pure Green vs. Brown Teams of Stakeholders

The team formation among stakeholders (excluding firms) is very simple in this setting. Due to the benefits of concentrating green stakeholders at the same teams (according to Lemma 1), green (brown) stakeholders are only matched with the green (brown) stakeholders of other types. That is, the market is fully segmented into purely brown vs green teams or firms. Moreover, there is a positive sorting between $x_\ell \, \forall \ell \leq N$ for each market, according to Lemma 2.

The full segmentation outcome thus implies that the team composition of stakeholders is the same as the shareholder-capitalism benchmark. Intuitively, since stakeholders only match (i.e., compete) with the ones with the same green preference, the ranking of stakeholders in the green (brown) market remains the same under Assumption 2. In this sense, there is no distortion in the matches among stakeholders.

Green teams target less productive firms. There is, however, distortion when matching with firms. According to Lemma 2, the matching between firms and the team of stakeholders can thus be characterized by positive sorting between firm productivity x_{N+1} and the z-index of the team. Given that the green team will have a lower index relative to an equivalent brown team, which means that green (brown) teams will be matched with firms that are less (more) productive relative to the benchmark.

Figure 1 illustrates this outcome, where the x-axis (y-axis) represents the productivity of the firm (their matching team). The productivity ranking of the team is given by $y_{-(N+1)}[i] = \prod_{\ell=1}^{N} x_\ell[i]$ under PAM. We further use a green (brown) line to represent the green (brown) team. Given y_{-N+1},the team then has green index of $n_{-(N+1)} = N$ ($n_{-(N+1)} = 0$) with probability λ $(1-\lambda)$.

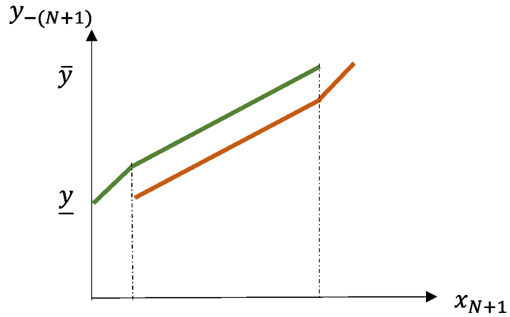

Figure 1: Matching between firm (x_{N+1}), and pure green ($y_{-(N+1)}, N$) and brown team ($y_{-(N+1)}, 0$) of stakeholders

Proposition 1. *(Full Segmentation) Under Assumption 2 and balanced supply* $\lambda_\ell = \lambda \ \forall \ell \leq N$*. Stakeholders with the same green preference are matched, and there is a positive sorting between* $x_\ell \ \forall \ell \in \{1, ..N\}$*. The matching between the teams and firms is characterized by positive sorting between* x_{N+1} *and index* $z(y_{-N+1}, n_{-N+1})$*.*

This simple case highlights that green stakeholders will get worse matches if no productive green counterparty is available. Specifically, in this case, since they can find a green stakeholder of other types but no green firms, it thus matches with a less productive firm. By the same logic, the distortion for the green team will go away if firms have green preference themselves (i.e., $\lambda_{N+1} = \lambda$), as firms and stakeholders are completely symmetric.[4]

4.2 Greenium

Earning premium for brown team. We now examine the earnings difference across (pure) brown and green team, conditional on the productivity of the team. Let $P^*(y, n)$ denote the total compensation received by the team of stakeholder with productivity y and green index

[4]It is worth noting that we assume the firm (i.e., insider) does have green preference themselves to highlight that firms are purely profit-maximizing (as in the classical setting), so their incentives for abatement only comes from their (endogenous) composition of shareholders. In general, our setup can also handle the case when firms have a green preference by relabeling firms as one of the stakeholders.

n. The earnings premium for the brown team yields

$$
\begin{aligned}
P^*(y,0) - P^*(y,N) &= P^*(y,0) - \{P^*((1-c\sigma)y,0) + c\xi_N^*\} \\
&= \int_{(1-c\sigma)y}^{y} x_{-N+1}^*(y,0)d\tilde{y} - c\xi_N^* \\
&= \underbrace{\int_{(1-c\sigma)y}^{y} \left\{x_{-N+1}^*(\tilde{y},0) - x_{-N+1}^*((1-c\sigma)y,0)\right\} d\tilde{y}}_{sorting\ effect} + \underbrace{c\left\{\sigma y x_{-N+1}^*(y,N) - \xi_N^*\right\}}_{cleaning\ costs},
\end{aligned}
$$

where the first equality uses the fact that a firm x_{N+1} must be indifferent between hiring a green with productivity y and a brown team with the same z with productivity $y' = (1-c\sigma)y$.[5] The second inequality uses the fact that for the brown team, we have

$$
P^*(y,0) = \max_{x_{N+1}} \Omega(yx_{N+1},0) - U^*(x_{N+1})
$$

and thus $\frac{\partial P(y,0)}{\partial y} = x_{-N+1}^*(y,0)$.

The last equality decomposes the premium into two terms. The first term captures that since (brown) green team is targeting (more) less productive firms. This term is zero if and only if firms are homogeneous. The second term represents the abatement cost for the firm that hires the green team.

This result highlights that the earning premium for the brown team is beyond the standard compensation differential since the brown team will work for a larger firm in equilibrium. The same logic holds for individual premiums. If all stakeholders are symmetric, the individual premium will be $\frac{1}{N}$ of the team premium.

4.3 Implications

We now examine the effect of increasing λ (i.e., all stakeholders become greener).

[5] That is, $U_{N+1}(x_{N+1}) = (1-c\sigma)yx_{N+1} + c\xi_N^* - N\psi(\xi_N^*) - \{P^*(y,N) - N\psi(\xi_N^*)\} = y'x_{N+1} - P^*(y',0)$.

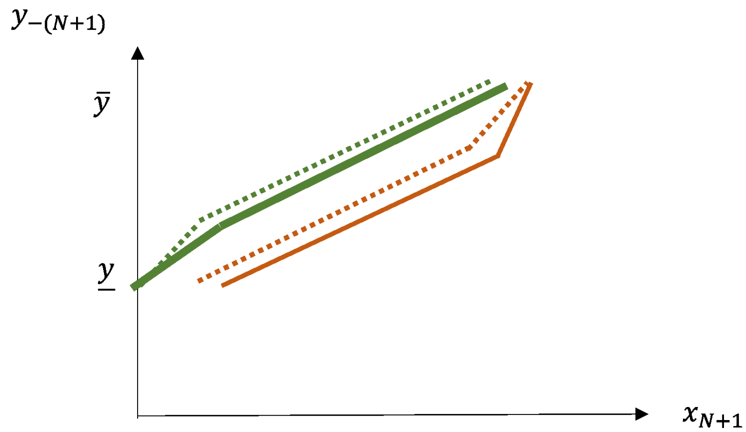

Figure 2: Reallocation effect of increasing the measure of green stakeholders.

Impact on extensive margin. Clearly, firms abate only if they hire a green team. Since the measure of firms hiring green teams is given by λ under the symmetric benchmark. This immediately implies that an increase in λ increases the measure of green firms, who will have emission level ξ_N^*. Thus, if we use the measure of green firms as the impact at the extensive margin. Then, there is a one-to-one increase in the impact with respect to λ.

Reallocation effect—remaining brown firms pollut more. On the other hand, this effect also changes the matching at the micro level. The solid (dashed) line in Figure 2 represents the matching after (before) increasing λ. An increase in the green team thus makes the brown team relatively scarce. As a result, the brown team will work for a more productive firm, which explains why the solid brown line shifts to the right relative to the dashed brown line.

Observe that total emissions can be expressed as

$$E = \lambda \xi_N^* + (1-\lambda) \int \left\{ \sigma y x_{N+1}^*(y, 0) \right\} dG(y), \tag{11}$$

where let $G(y)$ denote the cdf of the team with productivity lower than y.[6] That is, the first (second) term is the emission from green (brown) team. This shows two effects: a higher λ lowers emission as more firms abate. On the other hand, it also makes the brown team browner as higher λ also increases $x_{N+1}^*(y, 0)$.

[6]The $G(y)$ can be derived from PAM, as $y_{-N+1}[i] = \prod_{\ell=1}^{N} x_\ell[i]$.

Exit by the new green team. We now look at the effect from the viewpoint of the team that was brown but became green. The matching for these teams were represented by the dashed brown line but now switches to the solid green line. In other word, these teams will now work with less productive firms, which thus looks like they exited their original firm, which is brown but also more productive.

It is worth highlighting that the green team could potentially pay for abatement costs to pressure his original firms to green (i.e., engaging with their firms). This is because working with a more productive firm is beneficial as it implies a higher the surplus, as $(1-c\sigma)yx$ increase in firms' productivity after the abatement cost.[7] However, whether it's optimal for the green team to do so crucially depends on the equilibrium cost of keeping these firms. Intuitively, these firms can do better by matching with brown teams, which makes engaging relatively expensive.

Counterfactual costs for "engaging" large firms. To see this formally, let $\hat{P}\left((y,N),\hat{x}_{N+1}\right)$ denote the total compensation (i.e., price) received by the green team (y,N) when matching with his original firm $\hat{x}_{N+1} > x^*_{N+1}(y,N)$, which is larger than his new firm. The difference between $\hat{P}\left((y,N),\hat{x}_{N+1}\right)$ and the green team's equilibrium compensation $P^*(y,N)$ can be rewritten as the change in the surplus and the difference in firms' required profits in equilibrium $U_{N+1}(\hat{x}_{N+1}) - U_{N+1}(x^*_{N+1}(y,N))$, which yields

$$
\begin{aligned}
&\hat{P}\left((y,N),\hat{x}_{N+1}\right) - P^*(y,N) \\
=&(1-c\sigma)y\left\{\hat{x}_{N+1} - x^*_{N+1}(y,N)\right\} - \left\{U_{N+1}(\hat{x}_{N+1}) - U_{N+1}(x^*_{N+1}(y,N))\right\} \\
=&(1-c\sigma)y\left\{\hat{x}_{N+1} - x^*_{N+1}(y,N)\right\} - \int_{x^*_{N+1}(y,N)}^{\hat{x}_{N+1}} z^*_{-(N+1)}(\tilde{x}_{N+1})d\tilde{x}_{N+1} \leq 0.
\end{aligned}
$$

The inequality uses the fact that the more productive firms are matched with teams with higher index. That is, $z^*_{-(N+1)}(\tilde{x}_{N+1}) \geq (1-c\sigma)y\ \forall \tilde{x}_{N+1} > x^*_{N+1}(y,N)$. This expression shows that exactly because of the existence of brown teams - that have higher z-index (despite of being less productive) – makes engaging more productive firms too expensive. Hence, it is

[7] The expression uses the fact that the emission within the firm only depends on the green index, ξ^*_N; hence, the difference in the abatement is the difference in emission.

optimal for them to move to a new firm that is less productive.

4.4 Connection to Empirical Findings

Our symmetric benchmark illustrates key economic forces in our model. It also captures a couple of salient facts regarding the impact of green stakeholders on impact. The first is that the remaining brown firms that experience exit get browner (Kacperczyk and Peydró 2022). The second is that there is a positive greenium when it comes to stakeholder impact (Hong and Shore 2023).

However, this setting can not explain some other empirical facts.

1. The symmetric-impact setting generates only two levels of emissions intensity (emissions holding fixed revenue) — pure green or pure brown. But firm emissions intensity is highly dispersed, even controlling for firms in the same industry (see Figure 6). As such, we want a model that can deliver not just purely brown vs green teams of stakeholders, but also teams with a mix of both.

2. The earnings premium are often differ across different stakeholders. For instance, Kacperczyk and Peydró 2022 find small greeniums for for the 7% of green loans held by banks. In contrast, Krueger, Metzger, and Wu 2021 find large greeniums for workers at green firms, which account for around 25% of the workforce. We want a model that can match these facts.

3. The impact of stakeholders may differ across types, and some of them might have limited impact in the sense that they do not necessarily turn more brown firms to be green (Kacperczyk and Peydró 2022).

4. Our model currently only generates that exit is optimal. Green stakeholders have to exit to smaller firms and turn them green. But engagement, i.e. green stakeholders remaining at their firms and turning them greener, also occurs in practice.

5 Asymmetric Multistakeholder Impact

To rationalize these empirical findings in Section 4.4, we now allow for stakeholders to have different measure λ_ℓ, to capture the idea that certain groups of stakeholders are more scarce than others. Two new forces arise in the asymmetric setting.

Competition across preferences. When $\lambda_\ell > \lambda_{\ell'}$, it means that there are not enough skilled brown stakeholders of type ℓ to match with brown stakeholders of type ℓ'. Hence, a brown stakeholder of type ℓ' could potentially do better by competing for a more skilled green stakeholder ℓ. Thus, the competition for green stakeholders of type ℓ becomes stronger, which makes it harder for green stakeholders of type ℓ' to keep skilled green stakeholders of type ℓ. Such a channel is shut down under the symmetric case ($\lambda_\ell = \lambda_{\ell'}$), as they can always find a green stakeholder with the same skill. Hence, there is no need for stakeholders to match across preferences.

Brown stakeholders do not necessarily have a higher ranking. Second, since the supply of green stakeholders is not the same across groups, it thus means that certain brown stakeholders must be matched with some green stakeholders. Observe from Equation 10, if a brown stakeholder ends up teaming up with any green stakeholder, then his team is now subject to the discount $(1 - c\sigma)$. In other words, not all brown stakeholders can stay in a pure brown team, and thus not all of them can have a higher ranking relative to their green counterparts. The question is thus *which* brown stakeholders will have to match with green stakeholders?

5.1 Illustrative Example with $N = 2$

We now illustrate the key idea of constructing a stable match among stakeholders using a simple example with $N = 2$ and uniform distribution, where $\lambda_\ell > \lambda_{\ell'}$. The solid lines in Figure 3 represent the stable matches among stakeholder ℓ and ℓ' when $\lambda_\ell > \lambda_{\ell'}$. This graph can be separated intwo three regions.

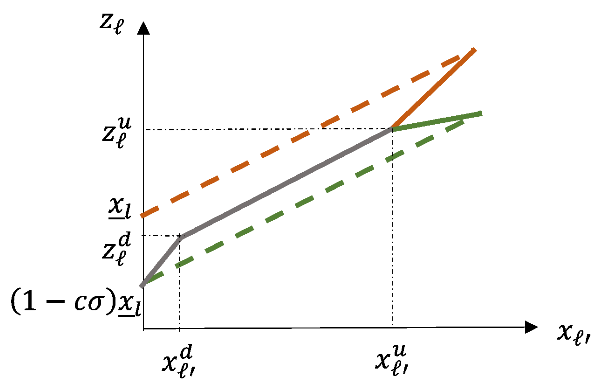

Figure 3: Separation on the top: The dash green (brown) line represents the z-index of the matching partner under the balanced supply for green (brown) stakeholder of type ℓ'.

(1) *Segmented market at the top of skills distribution.* ($x_{\ell'} > x_{\ell'}^u$): At the top skills region, i.e. high values of z_l and $x_{l'}$, the solid brown and green line represent the matching outcomes where stakeholder (x_ℓ, θ) only hires the stakeholders of the same green preference. Positive sorting on productivity holds for each market. Intuitively, since the top brown stakeholders of type ℓ' have higher incentives to avoid abatement, they will compete for the most skilled brown stakeholders of type ℓ.

However, since there are not enough productive brown stakeholders of type ℓ out there (since $\lambda_\ell > \lambda_{\ell'}$), the second tier of brown stakeholders of type ℓ' can only match with a very unproductive brown stakeholder of type l by staying in the brown market. Indeed, as there are relatively more (fewer) brown (green) stakeholders of type ℓ, the brown line goes down faster then the green line,[8] capturing the idea that the productivity of the brown (green) stakeholder of type ℓ decreases (increases) relative to the symmetric benchmark. This region thus ends when these two lines intersect, which represents that it is optimal for the brown stakeholder to start considering the green stakeholders as well, as they now have a higher z-index.[9]

(2) *Competition across preferences* ($x_{\ell'} \leq x_{\ell'}^u$): In this region, green stakeholders of type ℓ can be hired by either green and brown stakeholders of type ℓ'. The matching can be characterized as positive sorting between z_ℓ and $x_{\ell'}$, which is represented by the grey line (denoted by

[8]Formally, when $\lambda_\ell > \lambda_{\ell'}$,the slop of the brown is given by $\frac{(1-\lambda_{\ell'})}{(1-\lambda_\ell)} \frac{d\phi_\ell^B(x_{\ell'})}{dx_{\ell'}} > \frac{\lambda_{\ell'}}{\lambda_\ell} \frac{d\phi_\ell^B(x_{\ell'})}{dx_{\ell'}}$, which is larger than the slope of the green line.

[9]We are interested in the case where this cutoff exists; otherwise, one can show that it will be full separation expect at the lowest ability.

$\phi^*_\ell(x_{\ell'})$). The slope of the grey line is in between the green and brown dash line. This highlights the fact that agents with different green preferences are now competing in the same market.[10]

Let $\Psi_\ell(z_\ell)$ represent the corresponding measure of stakeholder ℓ with effective productivity index weakly below z_ℓ. The matching function $\phi^*_{\ell'}(x_\ell)$ thus solves

$$\Psi_\ell(\phi^*_{\ell'}(x_\ell)) = F_{\ell'}(x_{\ell'}). \tag{12}$$

In general, conditional on the same z_ℓ, stakeholders on the same teams could have different green preferences. According to Lemma 1, conditional on $(z_\ell, x_{\ell'})$, the green (brown) stakeholder of type ℓ is always first allocated to green (brown) stakeholder of type ℓ', conditional on $(z_\ell, x_{\ell'})$.

(3) *All green teams for low skilled stakeholders.* There exists a region at the bottom of $\Psi_\ell(z_\ell)$, where all stakeholders of type ℓ are all green. Thus, all stakeholders of $x_{\ell'}$ must be matched with green stakeholders of type ℓ only. This explains why the slope of the grey line is steeper at the bottom, as ℓ' type of stakeholders of both preferences are sharing the green stakeholders of type ℓ.

Which brown stakeholders can stay in a pure brown team? The green (brown) line in Figure 4 shows the probability of having a green counterparty as a function of stakeholder skill conditional on having green (brown) preference. Not surprisingly, stakeholders that are relatively scarce – i.e, green (brown) stakeholders of type ℓ' (of type ℓ) – do not need to match across preferences, which explains the probability of having a green partner is always one (zero) for green stakeholder of type ℓ' (of type ℓ).[11] The remaining stakeholders must be matched with stakeholders with a different green preference.

[10]The brown dashed line is thus given by $\phi^B_\ell(x_{\ell'})$, which can be solved in the standard way and $\frac{d\phi^B_\ell(x_{\ell'})}{dx_{\ell'}} = \frac{f_{\ell'}(x_{\ell'})}{f_\ell(\phi^B_\ell(x_{\ell'}))}$. That is, the slope is given by the relative mass of stakeholders at $x_{\ell'}$ and of his matching partner $\phi^B_\ell(x_{\ell'})$, which is constant under uniform distribution. The green stakeholder is also matching with the same ability $\phi^B_\ell(x_{\ell'})$ but subject to the discount $(1 - c\sigma)$. That is, the green line is given by $(1 - c\sigma)\phi^B_\ell(x_{\ell'})$.

[11]Specifically, the construction above ensures that stakeholders on the short side, which are green (brown) stakeholder of type ℓ' (of type ℓ), always matched with a partner with the same preference. Specifically, in the mixing region, the green stakeholder of type ℓ', which is relatively scarce, has priority to match with an agent with index $\phi^*_\ell(x_{\ell'})$ that are green. Since there are relatively more green agents of type ℓ by construction, this ensures that all green stakeholders of type ℓ' become greener after the match. Equivalently, the brown

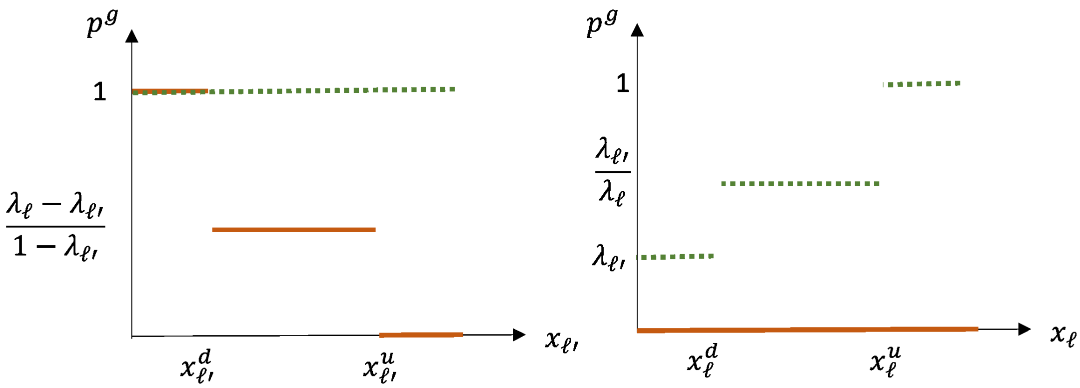

Figure 4: Probability of having a green counterparty, where type ℓ have excess green stakeholders relative to type ℓ' ($\lambda_\ell > \lambda_{\ell'}$).

High-skilled stakeholders do not mix. Full separation at the top of the skills distribution immediately suggests that the top brown stakeholder of type ℓ' must remain brown after matches. Intuitively, this is because that the benefit of avoiding abatement is highest at the top of the skills distribution. This also means that green stakeholder of type ℓ' on the top gets relatively good green match, as the equivalent brown firms will not compete with them.

Balancing productivity loss vs. abatement. Some brown stakeholders of type ℓ' at the bottom of the skills distribution would thus be matched to green stakeholders. Indeed, the probability of matching with a green stakeholder of type ℓ decreases with $x_{\ell'}$ for brown stakeholder of type ℓ'. This also explains why, from the viewpoint green stakeholder of type ℓ, the probability of matching a green stakeholder increases with x_ℓ.

However, Figure 4 also shows not all "excess" green stakeholders go to the bottom of the brown stakeholders, which would otherwise correspond to the case where the probability of matching with green stakeholder is one (zero) when the ability is below (above) a cutoff. Such a rule is not optimal because it would create unproductive matches for the middle brown stakeholders. Indeed, the existence of interior probability maps to the mixing region, where these brown stakeholders also match with green stakeholders that are relatively productive.

Our result thus highlights that the benefit of avoiding abatement is limited by the loss of productivity. The equilibrium allocation must then balance out these two effects.

stakeholder of type ℓ is also matched with the brown stakeholder with probability 1.

Lemma 3. *(Mixing Properties): (a) No mixing for stakeholders with relatively high ability. (b) When $\lambda_\ell > \lambda_{\ell'}$, all green (brown) stakeholders of type ℓ' (of type ℓ) is matched with a green (brown) partner. (c) When a green (brown) stakeholder mixes, he will be mixing between stakeholders with the same ability (z-index) but different green preferences.*

5.2 Sequential algorithm

We now generalize the construction above for general N. The basic idea of the general characterization is as follows. We construct the matching outcome by forming the team in a sequential order, starting from the stakeholders with the highest measure of green stakeholders $\lambda_1 \geq \lambda_2 ... \geq \lambda_N$.

We first form a team between stakeholders of type 1 and type 2 at period 1. The solution is given by the case with $N = 2$, where ℓ is the type 1 stakeholder and ℓ' is the type 2 stakeholder. Note that, in Figure 3, the type of stakeholders ℓ' that are on the $x-$axis are the ones with excess supply of *brown* stakeholders (i.e., type 2 in this case). Since the z-index is the relevant ranking for these brown stakeholders, our construction guarantees that it is indeed optimal for them to mix between stakeholders with the same z_ℓ. That is why showing the $z-$index of their matching counterparty on the $y-$axis is useful.

Once their matches are formed, we then treat them as a team that consists of types 1 and 2. Then, we solve for the bilateral matching between the team and the stakeholder of type 3 at period 2. By construction, the type 3 has excess brown stakeholders relative to the team, one can then construct the matching using the same graph where type 3 stakeholders are on the $x-$axis, and the relevant index on the $y-$axis is the z index of the team that consists of type 1 and 2. After they are matched, they will then become a team with 3 types of stakeholders, and then we solve for the matching outcome between this team with the stakeholder of type 4 using the same method.

By repeating this process, we will then form a team of N types of stakeholders. Finally, since the firms, by construction are all brown, the matching between firms and the team of stakeholders can simply be characterized by positive sorting between x_{N+1} and the one-dimensional index z of the team with N stakeholders.

The key difference from $N = 2$ is that the characteristics of the team at period τ endogenously depend on the matching outcome in the past. Formally, for each period τ, each team S_τ consists of agents of type $\ell \in \{1, 2, ..\tau\}$ consists of agents of type $\ell \in \{1, 2, ..\tau\}$ at period τ and can be characterized by (y_τ, n_τ). Let $\Psi_\tau(z)$ represent the corresponding measure of the team with an effective productivity index weakly below z at period τ, given the distribution of (y_τ, n_τ). Let $x^*_{\tau+1}(S_\tau)$ and $\theta^*_{\tau+1}(S_\tau)$ denote the characteristic of the matching stakeholder of type $\tau + 1$ for the team S_τ. The team productivity and green index of S_τ after matching with stakeholder $\tau + 1$ is thus given by $y_{\tau+1} = y_\tau x^*_{\tau+1}(S_\tau)$ and $n_{\tau+1} = n_\tau + \theta^*_{\tau+1}(S_\tau)$,respectively.

Proposition 2. *Under Assumption 2, the multilateral matching outcome can be characterized by the following sequential bilateral matching outcomes. For each period τ, the matching between the team S_τ and stakeholders $(x_{\tau+1}, \theta)$ can be characterized by a cutoff $x^u_{\tau+1}$ such that (1) for $x_{\tau+1} > x^u_{\tau+1}$, green (brown) stakeholders are only matched to the team with $n = \tau$ $(n = 0)$; (2) for $x_{\tau+1} \leq x^u_{\tau+1}$, the sorting is characterized by PAM between z-index of the team z_τ and the stakeholder ability $x_{\tau+1}$,where (a) the assignment function $\phi^*_\tau(x_{\tau+1})$ must satisfy*

$$\Psi_\tau(\phi^*_\tau(x_{\tau+1})) = F_{\tau+1}(x_{\tau+1})$$

and (b) conditional on the match $(z_\tau, x_{\tau+1})$,the green stakeholders $(x_{\tau+1}, 1)$ is allocated to the team with a higher green index first.

Evolution of green index. Observe that under this sequential process, the brown team at period τ (i.e, $n_\tau = 0$) will stay brown at period $\tau + 1$ and throughout the remaining sequential process. This is because, by construction, we add stakeholder types that are "browner" in later periods. Hence, the brown team can remain brown after $\tau + 1$. This also implies that not all green teams become greener after the match, as stakeholders in later periods has relative scarce green stakeholders. It also implies that the measure of brown team $n = 0$ is $(1 - \lambda_1)$, which is the measure of brown stakeholder of type 1.

5.3 Transfers

Since stakeholders are no longer symmetric, we look at the premium at the individual level. Given the equilibrium matches, Equation 3 implies that $\frac{\partial U_\ell(x_\ell,\theta)}{\partial x_\ell} = z^*_{-\ell}(x_\ell, \theta)$. Thus, the marginal gain for the ability x_ℓ depends on their marginal contribution to the surplus, which depends on the effective productivity (i.e., z-index) of his matching team. Specifically, unless it's a purely brown team ($n = 0$), the marginal value of x_ℓ will be discount by $1 - c\sigma$. The utility of the stakeholder (x_ℓ, θ) is uniquely pinned down,

$$U_\ell(x_\ell, \theta) = \int_{\underline{x}_\ell}^{x} z^*_{-\ell}(\tilde{x}_\ell, \theta) d\tilde{x}_\ell + U_\ell(\underline{x}_\ell, \theta),$$

given any $U_\ell(\underline{x}_\ell, \theta)$, which represents the utility of the stakeholder with the lowest ability. The compensation is thus uniquely pinned down for the brown stakeholders, as their equilibrium utility is simply their earning, $p_\ell(x_\ell, 0) = U_\ell(x_\ell, 0)$. Clearly, brown stakeholders $(x_\ell, 0)$ must receive the same fee despite the fact that he might work for teams with different green indexes in equilibrium.

On the other hand, green stakeholders care about the fee as well as the emission of the firm, which depends on the green index of the matching team. We thus let $p_\ell(x_\ell, 1|n)$ denote the fee when the stakeholder in a team with index n. We thus have

$$p_\ell(x_\ell, 1|n) = U_\ell(x_\ell, 1) + \psi(\xi^*_n).$$

Hence, if a green stakeholder is indifferent between working in firm different index n and n', then the difference in fees simply can be explained by the standard compensating differential, where $p_\ell(x_\ell, 1|n) - p_\ell(x_\ell, 1|n') = \psi(\xi^*_n) - \psi(\xi^*_{n'})$.

Only some stakeholders earn higher brown premium. Recall that, under the symmetric case, all brown stakeholders receive additional premiums relative to the standard compensating differential. This is because they are more valuable to a firm. However, if $(x_{\ell'}, 0)$ and $(x_{\ell'}, 1)$ end up matching with the team with the same index, then they will end up with the same ranking. For example, the stakeholders of type ℓ' in Figure 3 are the type that have relatively abundant brown stakeholders. The equilibrium predicts that, in this mixing region, this type of stakeholders will be matched with the same team $z^*_{-\ell}(x_\ell, 1) = z^*_{-\ell}(x_\ell, 0)$ regardless of their green preferences. Since there are relatively more green teams of $z_{-\ell}$, some brown stakeholders $(x_\ell, 0)$ will match with a green team. In this case, a green team $z_{-\ell}$ will be indifferent between hiring $(x_\ell, 1)$ and $(x_\ell, 0)$. Hence, a brown stakeholder $(x_\ell, 0)$ in the mixing region will not earn additional rent relative to an otherwise identical green stakeholder other than the standard compensating differential.

Proposition 3. *The earning premium for all brown stakeholders of type* 1 *(the type with the highest λ) has a positive ranking effect. The ranking effect is zero for stakeholder $(x_\ell, 0)$ where $x_\ell \leq x^u_\ell$.*

5.4 Implications

Impact on intensive margin only. Clearly, the measure of the pure brown firm $(n = 0)$ is given by $(1 - \lambda_1)$, which only depends on the lowest measure of brown stakeholders. Thus, increasing the measure of green stakeholders of type ℓ where $\ell > 1$ will not have any impact on the measure of brown firms. Their impact, however, is at the intensive margin as they will make green firms greener.

To fix the idea, consider the case where $N = 2$, where stakeholders are workers (type 1) and banks (type 2). In this example, we assume banks were all brown, but some of them are subject to green mandate and thus become green, while it is still the case that there is a higher measure of green workers (i.e., $\lambda_1 > \lambda_2$). [12]

[12]This is motivated by is the IBM Survey of 14,000 households, where 33% took a green job for 28% lower pay (see also estimates from Krueger, Metzger, and Wu (2021)), while Kacperczyk and Peydró (2022) report

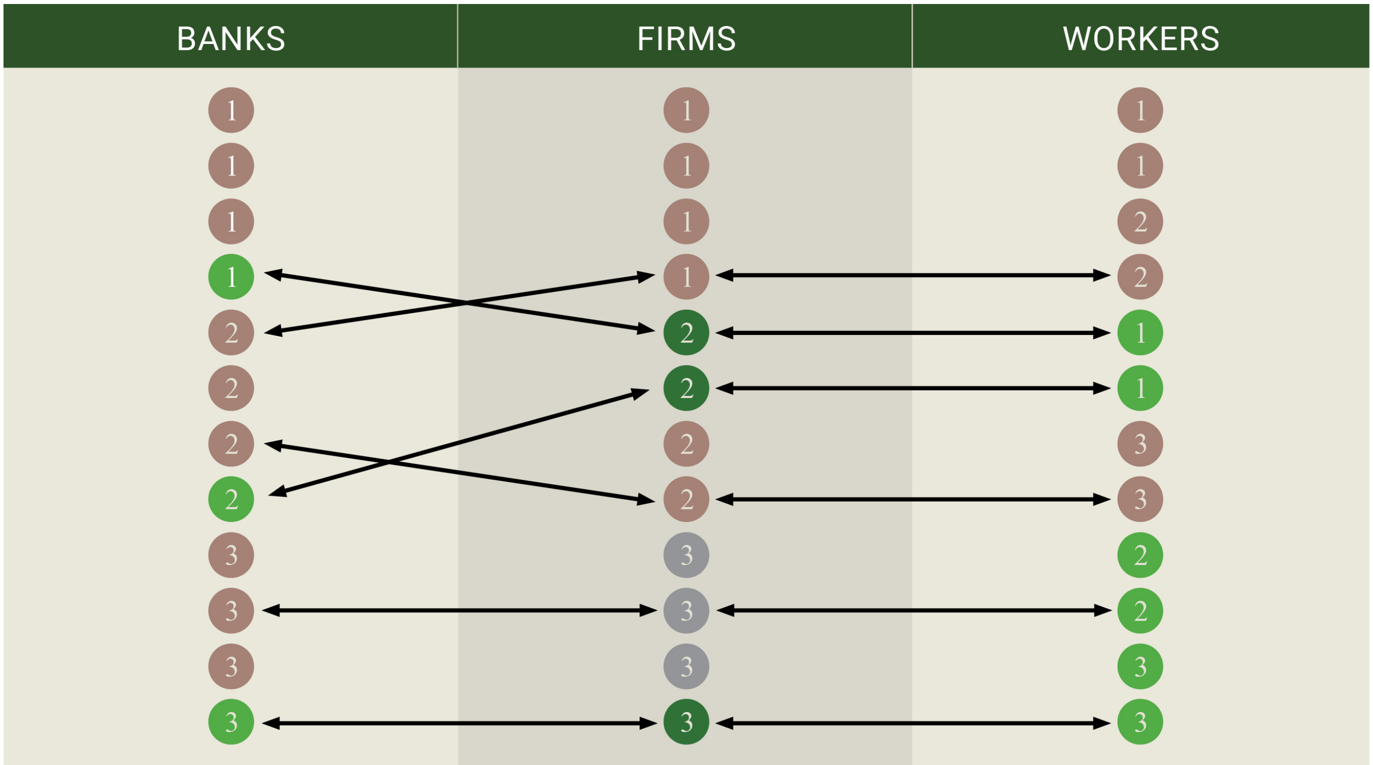

Figure 5: The effect of increasing the measure of green stakeholder of type 2 (i.e., banks)

Figure 5 illustrates the effect of introducing green banks. We use the grey (green) color to represent that the firm has one (two) green stakeholder in equilibrium. As discussed earlier, the measure of brown firms remains the same (which is 1/2, as there are 1/2 of brown workers); however, $\frac{1}{4}$ of firms become greener when λ_2 increases from 0 to $\frac{1}{4}$.

Exit vs engagement depends on firm size. This example also shows that, from the view point of banks that turn green, not all of them exit. Specifically, for the bank that were originally in the "grey" firm (i.e., the one with green worker), he will then simply make his existing firm greener, which thus looks like engaging with his existing firm. Intuitively, these are the banks that were already subject to the discount $(1 - c\sigma)$ because of matching with a green worker. Hence, making these banks green does not further decrease his ranking.

On the other hand, we also know that this can only happen for banks that are relatively small. This is because that all large brown banks must remain brown; therefore, if they become

that banks commit around 7% of their loans to only green firms.

green, they must go to a relative smaller firm, as we discussed in Section 4.3. In other words, our model predicts that, engagement (exit) is more likely to happen at the bottom (top) of the distribution.

6 A Test of Optimal Stakeholder Sorting

In this section, we provide evidence for the optimal sorting of green stakeholders. Ideally, if we have data on the latent productivity and preferences across multiple groups of stakeholders at a firm, we could directly test to our optimal teams predictions. Unfortunately, such data is not available.

6.1 Distribution of Number of Green Stakeholders

We propose an alternative test that is less data intensive. There are a large number of stakeholder groups, that are often categorized by whether they are internal or external. Internal stakeholders include employees, management, board members, shareholders. External stakeholders include customers, suppliers, government agencies, competitors, local communities, creditors, and media. Conservatively then, there are around 10 such groups, i.e. $N = 10$. Our model predicts that emissions, holding fixed firm revenue and industry, declines with the number of green stakeholders at a firm.

Hence, we can impute the number of green stakeholders for firms in a given industry by comparing their emissions intensity, the ratio of emissions to revenues. We denote the minimum and maximum emissions intensity by EI^{min} and EI^{max}. We then discretize the continuous interval between EI^{min} and EI^{max} into 10 equally spaced intervals. Firms are then assigned a value for the number of green stakeholders $GreenIndex$ depending on where their emissions intensity. By construction, the firms with values near EI_min are assigned a $GreenIndex$ equal to 10 and those near EI^{max} are assigned a $GreenIndex$ equal to 0.

Under a random-matching counterfactual, the realized $GreenIndex$ for a firm is shown to be distributed as a Binomial Poisson. To see why, consider a counterfactual environment where stakeholder's green preference are only realized after the match; hence, the matching are only based on the ability. The sorting on the green preference is thus shut down by construction, so the green preference within the match will be randomly. Given the realized green preference within the match, the firm then determines the abatement accordingly to maximize the joint surplus.

Since the matching is only based on ability (i.e., one-dimensional characteristic), the sorting can be simply characterized by standard positive sorting on each dimension x_ℓ. The nature of the random matching counterfactual implies that the realized green index n is characterize by the Poisson binomial distribution, where for each stakeholder type ℓ, the stakeholder has green preference with probability λ_ℓ.

The nature of the random matching counterfactual implies the following properties: Firm scale $y(\boldsymbol{x})$ and the green index n are independent, and, conditional any firm scale $y(\boldsymbol{x})$,the mean and the variance of green index is given by $\Sigma_{\ell=1}^N \lambda_\ell$ and $\Sigma_{\ell=1}^N \lambda_\ell(1-\lambda_\ell)$, respectively.

This property is in sharp contrast to our model's prediction. First of all, due to the concentration, conditional on firm size $y(\boldsymbol{x})$, the concentration of green preference imply that the probability of the green index at its extreme value $n \in \{0, N\}$ will be higher than the one predicted by the Poisson binomial distribution, while the Poisson binomial distribution predicts that the medium value of green index is more likely. In this sense, the our model predicts over-dispersion of green index relative to the random matching counterfactual.

6.2 Application to Utilities

We apply our test to utilities, firms in the GICS 5510 sector, that are one of the most carbon intensive sectors. The abatement decision in the context of power firms is whether they rely on renewables versus fossil fuels. State-level regulatory regime in the form of renewable portfolio standards play a role can be accounted for in our model by having local communities or regulators as one of the stakeholder groups.

Since this is a 4 digit GICS classification, we can be assured that firms are homogenous in

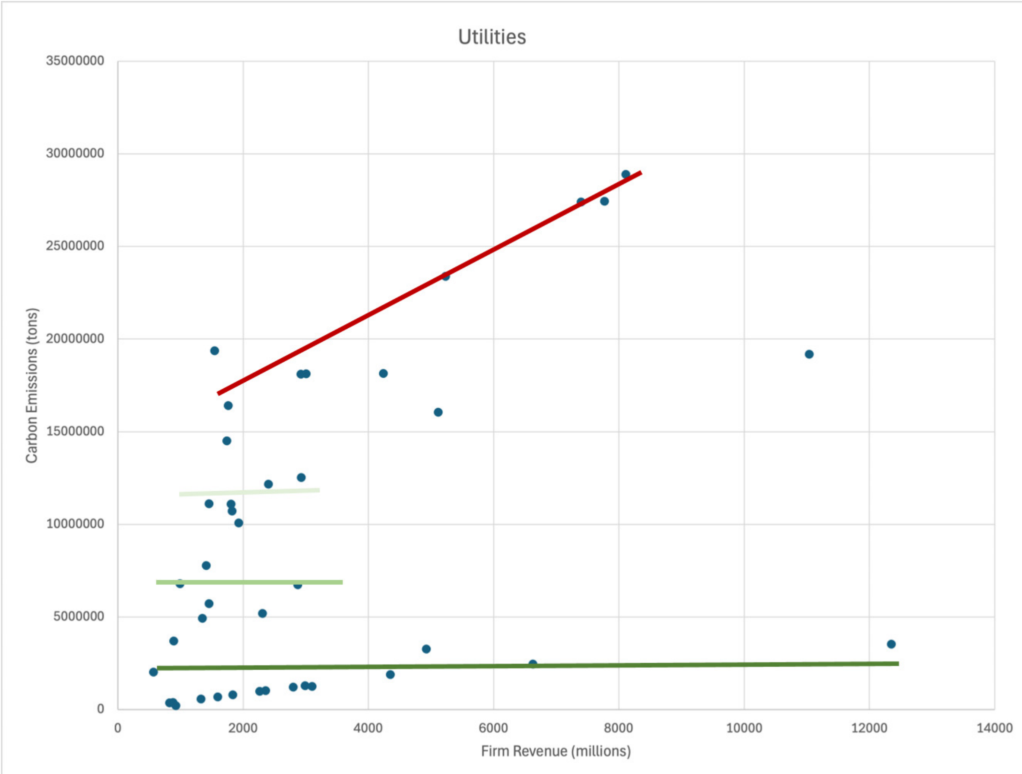

Figure 6: Utilities

terms of underlying businesses, i.e. these firms start with the same emissions-intensity σ but can endogenously adopt different abatement strategies depending on the allocation of green stakeholders.

The relationship is plotted for the utilities industry. We can see that there is more dispersion in emissions outcomes with firm revenue. We interpret this increasing dispersion with firm size as consistent with our model's prediction that there is bifurcation in terms of green and brown stakeholders for large firms. In contrast, for smaller firms, there is more a continuum of gray firms. Applying our test to this utilities data yields an overdispersion estimate of 1.5, rejecting the random sorting benchmark.

7 Conclusions

We develop a theory of how multiple groups of green stakeholders with preferences for reduced emissions impact their firms' abatement. Firms and stakeholders in competitive multi-sided markets sort into teams. We develop a new solution procedure to characterize the optimal formations. Optimal teams reflect both sorting on productivity due to output complementarities and sorting on green preferences due to sharing of abatement costs and the non-pecuniary benefits of reduced emissions.

Our model matches a number of salient facts that are difficult to reconcile using existing models. Either exit or engagement can be optimal, depending on firm size. Exit results in more green firms but leads the remaining brown firms to pollute more. The impact of green preferences on abatement varies across stakeholder groups, as do greeniums—the earnings of brown versus green stakeholders. We derive a test statistic based on emissions and revenue that supports stakeholder sorting.

References

Broccardo, Eleonora, Oliver Hart, Luigi Zingales, et al. (2022). "Exit versus voice". In: *Journal of Political Economy* 130.12, pp. 3101–3145.

Chiappori, Pierre-André, Robert McCann, and Brendan Pass (2016). "Multidimensional matching". In: *arXiv preprint arXiv:1604.05771*.

Chiappori, Pierre-André, Sonia Oreffice, and Climent Quintana-Domeque (2018). "Bidimensional matching with heterogeneous preferences: education and smoking in the marriage market". In: *Journal of the European Economic Association* 16.1, pp. 161–198.

Dupuy, Arnaud and Alfred Galichon (2014). "Personality traits and the marriage market". In: *Journal of Political Economy* 122.6, pp. 1271–1319.

Gabaix, Xavier and Augustin Landier (2008). "Why has CEO pay increased so much?" In: *The Quarterly Journal of Economics* 123.1, pp. 49–100.

Hatfield, John William and Scott Duke Kominers (2015). "Multilateral matching". In: *Journal of Economic Theory* 156. Computer Science and Economic Theory, pp. 175–206. ISSN: 0022-0531.

Heinkel, Robert, Alan Kraus, and Josef Zechner (2001). "The effect of green investment on corporate behavior". In: *Journal of financial and quantitative analysis* 36.4, pp. 431–449.

Hong, Harrison and Marcin Kacperczyk (2009). "The price of sin: The effects of social norms on markets". In: *Journal of financial economics* 93.1, pp. 15–36.

Hong, Harrison and Edward Shore (2023). "Corporate social responsibility". In: *Annual Review of Financial Economics* 15, pp. 327–350.

Kacperczyk, Marcin T and José-Luis Peydró (2022). "Carbon emissions and the bank-lending channel". In: *Available at SSRN 3915486*.

Krueger, Philipp, Daniel Metzger, and Jiaxin Wu (2021). "The sustainability wage gap". In: *Swedish House of Finance Research Paper* 20-14, pp. 21–17.

Lindenlaub, Ilse (2017). "Sorting multidimensional types: Theory and application". In: *The Review of Economic Studies* 84.2, pp. 718–789.

Pástor, L'uboš, Robert F Stambaugh, and Lucian A Taylor (2021). "Sustainable investing in equilibrium". In: *Journal of Financial Economics* 142.2, pp. 550–571.

Pedersen, Lasse Heje, Shaun Fitzgibbons, and Lukasz Pomorski (2021). "Responsible investing: The ESG-efficient frontier". In: *Journal of Financial Economics* 142.2, pp. 572–597.

Sattinger, Michael (1979). "Differential rents and the distribution of earnings". In: *Oxford Economic Papers* 31.1, pp. 60–71.

Tervio, Marko (2008). "The difference that CEOs make: An assignment model approach". In: *American Economic Review* 98.3, pp. 642–68.

A Appendix

A.1 Omitted Proofs

A.1.1 Proof for Lemma 1

Proof. Obverse that $\Omega(y,n) = \max_{e \leq \sigma y} \{y - n\psi(e) - c(\sigma y - e)\}$ is decreasing and convex in n. This is because that $f(n,e|y) \equiv y - n\psi(e) - c(\sigma y - e)$ is linear in n and thus $\Omega(y,n) = \max_e f(n,e|y)$ is (strictly) convex in n. This holds for a general cleaning cost function. With the specified linear cost, it can further be reduced to

$$\Omega(y,n) = (1 - c\sigma)y + \chi(n,y),$$

where $\chi(y,n) \equiv \max_{e \geq c\sigma y} \{ce - n\psi(e)\} = c\xi_n^* - n\psi(\xi_n^*)$ for any $n \geq 1$, and $\chi(y,0) = c\sigma y$. This further implies that $\chi_n < 0$ and $\chi(y,n)$ is convex in n ($\chi_{nn} > 0$). The property of $\chi(y,n)$ can be summarized by the Lemma below. [13]

Lemma 4. $\chi(y,n) - \chi(y,n+1)$ *decreases in* n $\forall n$, *and* $\chi(y,n) - \chi(y,n+1)$ *is independent of* y *for* $n \geq 1$, *and increasing in* y *only when* $n = 0$.

We now prove this result by contradiction. Suppose the the green index of the team for the green agent's $(x_i, 1)$ is lower that the one of an otherwise identical brown agent x_j, where $n_{-i} < n_{-j}$. We now show that the profitable deviation exists by switching their team. Intuitively, as both agents have the same ability, switching their teams do not affect the team productivity; however, since $\chi(y,n)$ is convex in n, switching results in more extreme value of n and thus increase total surplus.

[13] Our assumption 1 implies that it's optimal for any team to abate as long as there is one green stakeholder. More generally, similar properties hold as long as the interior solution exists for any $n \geq \hat{n}$. In this case, $\chi(y,n) = c\sigma y \; \forall n < \hat{n}$.

Formally, given $x_i = x_j = x$, the total surplus after switching yields

$$\begin{aligned}&\{\Omega(y_{-i}x_j, n_{-i}) + \Omega(y_{-j}x_i, n_{-j}+1)\} - \{\Omega(y_{-i}x_i, n_{-i}+1) + \Omega(y_{-j}x_j, n_{-j})\}\\ =&\{\chi(y_{-i}x_j, n_{-i}) + \chi(y_{-j}x_i, n_{-j}+1)\} - \{\chi(y_{-i}x_i, n_{-i}+1) + \chi(y_{-j}x_j, n_{-j})\} > 0\end{aligned}$$

where the inequality uses the fact when $n_j > n_i > 0$, Lemma 4 implies that

$$\chi(y_{-i}x, n_{-i}) - \chi(y_{-i}x, n_{-i}+1) > \chi(y_{-j}x, n_{-j}) - \chi(y_{-j}x, n_{-j}+1).$$

What is left to show is when $n_i = 0$,and $n_{-j} > 0$, in this case, we have

$$\begin{aligned}\chi(y_{-i}x, 0) - \chi(y_{-i}x, 1) &> \chi(y_{-i}x, n_{-j}) - \chi(y_{-i}x, n_{-j}+1)\\ &= \chi(y_{-j}x, n_{-j}) - \chi(y_{-i}x, n_j+1)\end{aligned}$$

□

A.1.2 Proof for Lemma 2

Proof. A stakeholder's problem can be rewritten as choosing his team optimally, by taking as given the composition of the team which consists of all types of stakeholders (excluding his own type) and the total equilibrium utilities of agents in the team, which is denoted by $\mathit{\Pi}(y_{-\ell}, n_{-\ell}) \equiv \Sigma_{\ell' \in L \setminus \{\ell\}} U(a_{\ell'})$. Hence, his optimization problem yields

$$U_\ell(x_\ell, \theta_\ell) = \max_{(y_{-\ell}, n_{-\ell})} \Omega(y_{-\ell}x_\ell, n_{-\ell}+\theta_\ell) - \mathit{\Pi}(y_{-\ell}, n_{-\ell})\Sigma_{\ell' \in L \setminus \{\ell\}} U(a_{\ell'}).$$

Since Equation 8 implies complementarity between green agent $(x_\ell, 1)$ and $y_{-\ell}$, hence, by the monotonic comparative statics, a green agent with higher ability must choose a team with a higher productivity than a green agent with lower ability. Similarly, Equation 9 implies complementarity between brown agent $(x_\ell, 0)$ and $z_{-\ell}$; hence, a more skilled brown agent must choose a team with a higher z-index. □

A.1.3 Proof for Proposition 1

Proof. Clearly, conditional on preference, PAM is the stable outcome due to the standard complementarity. Hence, what is left to show is that it is not optimal for agents to match across markets. Consider a green stakeholder $(x_i, 1)$ a brown stakeholder $(x_j, 0)$ switch teams, where $(y^*_{-i}, N-1)$ and $(y^*_{-j}, 0)$ represent their original team. The total surplus after switches yields

$$\begin{aligned}
&\Omega(y^*_{-i}x_j, N-1) + \Omega(y^*_{-j}x_i, 1) \\
=&(1-c\sigma)\left(y^*_{-i}x_j + y^*_{-j}x_i\right) + \chi(y^*_{-i}x_i, N-1) + \chi(y^*_{-j}x_j, 1) \\
<&(1-c\sigma)\left(y^*_{-i}x_j + y^*_{-j}x_i\right) + \chi(y^*_{-i}x_i, N) + \chi(y^*_{-j}x_j, 0) \\
=&\Omega(y^*_{-i}x_i, N) + \Omega(y^*_{-j}x_j, 0),
\end{aligned}$$

where the first equality uses the fact that $\chi(y, n)$ is independent of n for $n \geq 1$.

The second inequality uses the fact that, under PAM and full separation $y^*_{-i} \geq y^*_{-j}$ iff $x_i \geq x_j$. Hence, $\left(y^*_{-i}x_j + y^*_{-j}x_i\right) \leq y^*_{-i}x_i + y^*_{-j}x_j$; In other words, there is no productivity distortion in $(y_{-\ell}, x_\ell)$. That is, intuitively, there is no gain in aggregate surplus when matching across markets. There is, however, a cost of doing so, as $\chi(y, n)$ is convex in n, as we have for any $\hat{n} > 0$,according to Lemma 4,

$$\chi(y_j, 0) - \chi(y_j, 1) > \chi(y_j, N-1) - \chi(y_j, N) = \chi(y_i, N-1) - \chi(y_i, N).$$

$\square$

A.1.4 Proof for Proposition 2

Proof. Observe that the sequential ordering implies the following properties: (1) for any brown team at period τ, their team remains brown after matches. That is, if $n_\tau = 0$, then $n_{\tau+1} = 0$. Intuitively, this is because that the stakeholders at the later periods, by construction, are browner. (2) given any team $S_\tau =(y_\tau, n_\tau)$,where $n_\tau \geq 1$, we have $n_{\tau+1} = n_\tau + 1$ if $z_\tau(y_\tau, n_\tau) \geq$

$z^u_{\tau+1}$, and conditional on y_τ, $n_{\tau+1} = n_\tau + 1$ iff $n_\tau \geq \hat{n}_\tau$. This is because that, in the mixing regions, conditional on z_τ, only the relative green team get another green stakeholder.

As a result, no matter for green or brown team, the evolution of their z index can be expressed as $z_{\tau+1} = z_\tau x^*_{\tau+1}(S_\tau)$,[14] and $n_{\tau+1} = n_\tau + \theta^*_{\tau+1}(S_\tau)$. Let $X^*_\tau(S_\tau)$ and $N^*_\tau(S_\tau)$ represent the optimal productivity and green index chosen by the team S_τ from period τ to period N.

$$\Omega\left\{(y_\tau, n_\tau), (x_{\tau+1}, \theta_{\tau+1})\right\} = z_\tau(y_\tau, n_\tau)\, x_{\tau+1} X^*_{\tau+1}(S_{\tau+1}) + \chi\left(\left(n_\tau + \theta_{\tau+1} + N^*_{\tau+1}(S_{\tau+1})\right)\right).$$

Given that $X^*_{\tau+1}(S_{\tau+1})$ is monotonic in $z_{\tau+1}$ and $N^*_{\tau+1}(S_{\tau+1})$ is monotonic in $n_{\tau+1}$, it is sufficient to show that the matching outcome maximizes the product of $z_{\tau+1} = z_\tau x_{\tau+1}$ and the dispersion of $n_{\tau+1}$ at period τ. In other words, we now show that the matching is stable given any period τ. Since our construction satisfies Lemma 2, conditional on the preference, the matching is stable.

What is left to show is there is no profitable deviation for stakeholders to match across types. Consider first the case where a green stakeholder i considers to switch with a brown stakeholder j,whose team has green index $n_{-j} > 0$. That is, it must be the case where $x_j < x^d_\tau$

$$\begin{aligned}
&\tilde{\Omega}\left((y_{-i}, n_{-i}), (x_i, 1)\right) + \tilde{\Omega}\left((y_{-j}, n_{-j}), (x_j, 0)\right) \\
&- \left\{\tilde{\Omega}\left((y_{-i}, n_{-i}), (x_j, 0)\right) + \tilde{\Omega}\left((y_{-j}, n_{-j}), (x_i, 1)\right)\right\} \\
=&(1 - c\sigma)\left\{y_{-i}x_i + y_{-j}x_j - (y_{-i}x_j + y_{-j}x_i)\right\} + \left\{\chi(n_{-i}+1) + \chi(n_{-j}) - (\chi(n_{-i}) + \chi(n_{-j}+1))\right\} \geq 0,
\end{aligned}$$

The first term is positive, as by construction, $y_{-i} \geq y_{-j}$ iff $x_i \geq x_j$ given that $(x_j, 0)$ is in the mixing region. The second term is also positive as $\chi(n)$ is convex. Next, consider the case where $n_{-j} = 0$. In this case, the loss is even higher as both teams have to abate. $\square$

[14] Importantly, this is not true if property (1) does not hold. This is because that, if a brown team receives a green stakeholder a period τ', then $z_{\tau+1} = z_\tau(1 - c\sigma)x^*_{\tau+1}(S_\tau)$.

A.1.5 Proof for Proposition 3

Proof. Since our equilibrium implies that if $x_\ell < x_\ell^d$, then the brown stakeholder will be mixing between brown and green stakeholders with the team with same $z_{\tau-1}$ at period ℓ. As a result, they will have the same z_τ after the matches and since $\hat{x}_\tau$ increases in τ, the productivity of their sequential matching outcome remains the same. Hence, $z^*_{-\ell}(x_\ell, 0) = z^*_{-\ell}(x_\ell, 1)$. $\square$