Continuous Time Finance
Swedish House of Finance – Stockholm School of Economics
Summer 2024

teacher: Johan Waldén

schedule:

Tue, Aug 6,	9-11, 13-15
Wed, Aug 7,	9-11, 13-15
Thu, Aug 8,	9-11, 13-15
Fri, Aug 9,	9-11
Tue, Aug 13,	9-11, 13-15
Wed, Aug 14,	9-11, 13-15
Thu, Aug 15,	9-11, 13-15
Fri, Aug 16,	9-11

overview: The development of derivatives markets may be the single most important innovation in financial markets in the last fifty years. The celebrated Merton, Black & Scholes option pricing formula derived in the 70's has arguably been the most successful research paper in social sciences. The formula has been widely adopted by market participants, and has initiated a new field, Financial Engineering, which occupies researchers in financial institutions and universities throughout the world.

The focus of the course is on applied stochastic calculus applied to problems within finance. We will cover the theory of noarbitrage, Brownian motion, Ito integrals and calculus (Ito's lemma), change of measure (Girsanov's theorem), and links to partial differential equations (Feynman Kac's theorem, Kolmogorov equations). We apply the theory to derive many continuous time asset pricing formulas, including the celebrated Black-Scholes formulas for pricing plain-vanilla options. We also discuss equilibrium models in continuous time (Lucas tree economy, Cox Ingersoll Ross economy)

required text: Tomas Björk, "Arbitrage Theory in Continuous Time", 4th edition, 2020.

Johan Walden, "Quantitative Finance" notes, 2024.

optional texts: Steven Shreve, "Stochastic Calculus for Finance I: The Binomial Asset Pricing Model"

Steven Shreve, "Stochastic Calculus for Finance II: Continuous-Time Models".

Karatzas & Shreve, "Methods of Mathematical Finance".

Darrell Duffie, "Dynamic Asset Pricing Theory", 3rd edition.

attendance: Please attend all classes and do not arrive late.

grades: Pass, Pass with Distinction, or Fail. Grades will be based on:

Class participation	20.0 %
Assignment:	40.0 %
Final exam:	40.0 %
Total	100.0 %

final exam: There will be an open book take-home final exam that students should solve individually. The date for the exam is yet to be determined, but it will be sometime between August 20-30.

assignment: There will be one, quite extensive, assignment that students may solve individually or in groups of two (no more!). The assignment is due on Friday, August 23.

ethics and etiquette:

Students are allowed to consult all the material provided in the course (slides, lecture notes, course books, etc.), but are not allowed to use any external material that resembles a "solution" to an assignment. If there is any doubt, please contact the professor. Students are also allowed to discuss course material, including assignments, with each other. However, any help must stop far short of hinting (or providing) the solution to an assignment, in form of explicit calculations, etc. **The material in the course (slides, lecture notes, assignments, solutions, etc.) is proprietary. Posting of material on the internet is strictly prohibited and will lead to legal action.**

Course Outline:

Chapters refer to Walden (W) and Björk (B). This outline is tentative. Depending on the time constraints, we may not have time to cover everything on the list.

Discrete Model • One-period: noarbitrage, fundamental theorem of asset pricing, price systems, martingale pricing • Binomial model & general multi-period model	B: Chapters 2-3 W: Chapters 6-8
Continuous time modeling • Stochastic integrals • Relationship to differential equations • Dynamic portfolios • Arbitrage Pricing	B: Chapters 4-7 W: Chapters 9.1-9.5
The Black-Scholes model, numerical methods • Completeness, pricing, parity, the greeks, numerical methods	W: Chapters 9.5, 11 B: Chapters 8-10
Martingale pricing – Girsanov's theorem • The fundamental theorem in continuous time, stochastic discount factors, Girsanov's theorem	W: Chapter 9.6 B: Chapters 11-13
Dividends, Multidimensional Models • Pricing, risk-neutral valuation, state space reduction, exchange option, martingale approach	W: 10.1, 10.4 B: Chapter 14, 16
Change of numeraire • numeraire invariance, Girsanov transformations exchange option again	W: 10.5 B: Chapter 15
Term structure models • Short rate models, Affine term structure models	B: Chapter 19-21 W: Chapter 10.3
Equilibrium models in continuous time • *Lucas one-tree model, Cox Ingersoll Ross model*	W: Chapter 12